Title 7: Education K-12

Part 111

DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

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Background on the Dynamic Learning Maps Essential Elements

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach

the specified standard. Learning maps of this size had not been previously developed, and as a result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

Alignment of the DLM EEs to the DLM Learning Maps

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment. The teams of content experts most closely involved with this alignment work included:

Mathematics	**English Language Arts**
Kelli Thomas, Ph.D. (co-lead)	Caroline Mark, Ph.D. (lead)
Angela Broaddus, Ph.D. (co-lead)	Jonathan Schuster, Ph.D.
Perneet Sood	Russell Swinburne Romine, Ph.D.
Kristin Joannou	Suzanne Peterson
Bryan Candea Kromm	

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

The Alignment Process

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts.

This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

Claims and Conceptual Areas

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of

academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions

of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

Claim 1	**Number Sense: Students demonstrate increasingly complex understanding of number sense.** Conceptual Areas in the Dynamic Learning Map: **MC 1.1 Understand number structures (counting, place value, fraction)** *Essential Elements Included: K.CC.1.4, 5; 1.NBT.1a-b; 2.NBT.2a-b, 3; 3.NBT.1, 2, 3, 4,NBT.3; NF.1-3; 4.NF.1-2, 3; 5.NF.1, 2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; M.EE.8.NS.2.a* **MC 1.2 Compare, compose, and decompose numbers and sets** *Essential Elements Included: K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.1, 2; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.1-4* **MC 1.3 Calculate accurately and efficiently using simple arithmetic operations** *Essential Elements Included: 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5, 6-7; 6.NS.2, 3; 7.NS.1, 2a, 2b; 8.NS.1; 8.EE.1; HS.N-CN.2, 2.a, 2.b; HS.N-RN.1; HS.S-CP.1-5; HS.S-IC.1-22*
Claim 2	**Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.** Conceptual Areas in the Dynamic Learning Map: **MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes** *Essential Elements Included: K.MD.1; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2, 2a, 2b; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; HS.G-CO.1, 4-5, 6-8; HS.G-GMD.1-3, 4* **MC 2.2 Solve problems involving area, perimeter, and volume** *Essential Elements Included: 1.G.3; 3.G.2; 4.G.3; 4.MD.2; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; HS.G-GMD.1-3; HS.G-GPE.7*
Claim 3	**Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.** Conceptual Areas in the Dynamic Learning Map: **MC 3.1 Understand and use measurement principles and units of measure** *Essential Elements Included: 1.MD.1-2, 3a, 3b, 3c, 3d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2a, 2b, 2c, 2e; 5.MD.1a, 1b, 1c; HS.N-Q.1-3*

	MC 3.2 Represent and interpret data displays *Essential Elements Included: 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4a, 4b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; HS.S-ID.1-2, 3, 4*
Claim 4	**Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.** Conceptual Areas in the Dynamic Learning Map: **MC 4.1. Use operations and models to solve problems** *Essential Elements Included: K.OA.1, 1a, 1b, 2, 5a, 5b; 2.OA.1, 3, 4; 3.OA.1-2, 8; 4.OA.1-2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1-2, 4; 8.EE.7; HS.A-CED.1, 2-4; HS.A-SSE.1, 3* **MC 4.2 Understand patterns and functional thinking** *Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.3; 8.EE.5-6; 8.F.1-3, 4, 5; HS.A-REI.10-12; HS.A-SSE.4; HS.F-BF.1, 2; HS.F-IF.1-3, 4-6; HS.F-LE.1*

A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

Resulting Changes to the DLM Essential Elements

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language

arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding “With guidance and support” to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

Access to Instruction and Assessment

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student's physical inability to produce independent speech.

- alternate access devices that compensate for a student's physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

Guidance and Support

The authors of the CCSS use the words "prompting and support" at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade, encourage, or remind someone to do or say something" (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., “Tell me what to do first.”),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, “See if you can write the first letter in the word, /d/og [phonetically pronounced].”),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

Conclusion

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

Review of Draft Two of Dynamic Learning Maps Essential Elements

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

Development of the Original Dynamic Learning Maps Common Core Essential Elements

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who

led the original effort included:

Jan Sheinker, Sheinker Educational Services, Inc.
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DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR KINDERGARTEN

Kindergarten Mathematics Domain: Counting and Cardinality

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Know number names and the count sequence.	
K.CC.1. Count to 100 by ones and by tens.	EE.K.CC.1. Starting with one, count to 10 by ones.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Not applicable. See EE.2.NBT.2.b.
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Not applicable. See EE.2.NBT.3.
CLUSTER: Count to tell the number of objects.	
K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.	EE.K.CC.4. Demonstrate one-to-one correspondence, pairing each object with one and only one number and each number with one and only one object.
K.CC.4.a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.	
K.CC.4.b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	
K.CC.4.c. Understand that each successive number name refers to a quantity that is one larger.	
K.CC.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	EE.K.CC.5. Count out up to three objects from a larger set, pairing each object with one and only one number name to tell how many.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Compare numbers.	
K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]	EE.K.CC.6. Identify whether the number of objects in one group is more or less than (when the quantities are clearly different) or equal to the number of objects in another group.
K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.	Not applicable. See EE.2.NBT.4.

[1] Include groups with up to ten objects.

Kindergarten Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	
K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	EE.K.OA.1. Represent addition as "putting together" or subtraction as "taking from" in everyday activities.
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Not applicable See EE.2.NBT.6–7.
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Not applicable. See EE.1.NBT.6.
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	Not applicable. See EE.1.NBT.2.
K.OA.5. Fluently add and subtract within 5.	Not applicable. See EE.3.OA.4.

[2] Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Kindergarten Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Work with numbers 11–19 to gain foundations for place value.	
K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Not applicable. See EE.1.NBT.4 and EE.1.NBT.6.

Kindergarten Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Describe and compare measurable attributes.	
K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	EE.K.MD.1-3. Classify objects according to attributes (big/small, heavy/light).
K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children, and describe one child as taller/shorter.*	
CLUSTER: Classify objects and count the number of objects in each category.	
K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]	Not applicable. See EE.1.MD.4.

[3] Limit category counts to be less than or equal to 10.

Kindergarten Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	
K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.	Not applicable. See EE.1.G.a.
K.G.2. Correctly name shapes regardless of their orientations or overall size.	EE.K.G.2–3. Match shapes of same size and orientation (circle, square, rectangle, triangle).
K.G.3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
CLUSTER: Analyze, compare, create, and compose shapes.	
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	Not applicable. See EE.7.G.1.
K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Not applicable.
K.G.6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	Not applicable. See EE.1.G.3.

First Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving addition and subtraction.	
1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	EE.1.OA.1.a. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), or acting out situations.
	EE.1.OA.1.b. Recognize two groups that have the same or equal quantity.
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	EE.1.OA.2. Use "putting together" to solve problems with two sets.
CLUSTER: Understand and apply properties of operations and the relationship between addition and subtraction.	
1.OA.3. Apply properties of operations as strategies to add and subtract.[4] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a 10, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	Not applicable. See EE.6.EE.3 and EE.N-CN.2.
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	Not applicable. See EE.1.NBT.4 and EE.1.NBT.6.

[4] Students need not use formal terms for these properties.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Add and subtract within 20.	
1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	EE.1.OA.5.a. Use manipulatives or visual representations to indicate the number that results when adding one more.
	EE.1.OA.5.b. Apply knowledge of "one less" to subtract one from a number.
1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	Not applicable. See EE.3.OA.4.
CLUSTER: Work with addition and subtraction equations.	
1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	Not applicable. See EE.1.OA.1.b and EE.2.NBT.5.a.
1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*	Not applicable. See EE.3.OA.4.

First Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the counting sequence.	
1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals, and represent a number of objects with a written numeral.	EE.1.NBT.1.a. Count by ones to 30.
	EE.1.NBT.1.b. Count as many as 10 objects and represent the quantity with the corresponding numeral.
CLUSTER: Understand place value.	
1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	EE.1.NBT.2. Create sets of 10.
1.NBT.2.a. 10 can be thought of as a bundle of ten ones—called a "ten."	
1.NBT.2.b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	
1.NBT.2.c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	EE.1.NBT.3. Compare two groups of 10 or fewer items when the number of items in each group is similar.
CLUSTER: Use place value understanding and properties of operations to add and subtract.	
1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	EE.1.NBT.4. Compose numbers less than or equal to five in more than one way.

CCSS Grade-Level Standards	DLM Essential Elements
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Not applicable. See EE.1.OA.5.a and EE.1.OA.5.b.
1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	EE.1.NBT.6. Decompose numbers less than or equal to five in more than one way.

First Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Measure lengths indirectly and by iterating length units.	
1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	EE.1.MD.1–2. Compare lengths to identify which is longer/shorter, taller/shorter.
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	
CLUSTER: Tell and write time.	
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks.	EE.1.MD.3.a. Demonstrate an understanding of the terms *tomorrow*, *yesterday*, and *today*.
	EE.1.MD.3.b. Demonstrate an understanding of the terms *morning*, *afternoon*, *day*, and *night*.
	EE.1.MD.3.c. Identify activities that come before, next, and after.
	EE.1.MD.3.d. Demonstrate an understanding that telling time is the same every day.
CLUSTER: Represent and interpret data.	
1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	EE.1.MD.4. Organize data into categories by sorting.

First Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	EE.1.G.1. Identify the relative position of objects that are on, off, in, and out.
1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[5]	EE.1.G.2. Sort shapes of same size and orientation (circle, square, rectangle, triangle).
1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of* or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	EE.1.G.3. Put together two pieces to make a shape that relates to the whole (i.e., two semicircles to make a circle, two squares to make a rectangle).

[5] Students do not need to learn formal names such as “right rectangular prism.”

Second Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving addition and subtraction.	
2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable. See EE.3.OA.4.
CLUSTER: Add and subtract within 20.	
2.OA.2. Fluently add and subtract within 20 using mental strategies.[6] By end of Grade 2, know from memory all sums of two one-digit numbers.	Not applicable. See EE.2.NBT.6–7 and EE.3.OA.4.
CLUSTER: Word with equal groups of objects to gain foundations for multiplication.	
2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	EE.2.OA.3. Equally distribute even numbers of objects between two groups.
2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	EE.2.OA.4. Use addition to find the total number of objects arranged within equal groups up to a total of 10.

[6] See standard 1.OA.C.6 for a list of mental strategies.

Second Grade Mathematics: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand place value.	
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:	EE.2.NBT.1. Represent numbers up to 30 with sets of tens and ones using objects in columns or arrays.
2.NBT.1.a. 100 can be thought of as a bundle of ten tens—called a "hundred."	
2.NBT.1.b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.	EE.2.NBT.2.a. Count from 1 to 30 (count with meaning; cardinality).
	EE.2.NBT.2.b. Name the next number in a sequence between 1 and 10.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	EE.2.NBT.3. Identify numerals 1 to 30.
2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	EE.2.NBT.4. Compare sets of objects and numbers using appropriate vocabulary (more, less, equal).
CLUSTER: Use place value understanding and properties of operations to add and subtract.	
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	EE.2.NBT.5.a. Identify the meaning of the "+" sign (i.e., combine, plus, add), "–" sign (i.e., separate, subtract, take), and the "=" sign (equal).
	EE.2.NBT.5.b. Using concrete examples, compose and decompose numbers up to 10 in more than one way.

CCSS Grade-Level Standards	DLM Essential Elements
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.	EE.2.NBT.6-7. Use objects, representations, and numbers (0–20) to add and subtract.
2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	
2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Not applicable.
2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[7]	Not applicable.

[7] Explanations may be supported by drawings or objects.

Second Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Measure and estimate lengths in standard units.	
2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	EE.2.MD.1. Measure the length of objects using non-standard units.
2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Not applicable.
2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.	EE.2.MD.3–4. Order by length using non-standard units.
2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
CLUSTER: Relate addition and subtraction to length.	
2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	EE.2.MD.5. Increase or decrease length by adding or subtracting unit(s).
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	EE.2.MD.6. Use a number line to add one more unit of length.
CLUSTER: Work with time and money.	
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using *a.m.* and *p.m.*	EE.2.MD.7. Identify on a digital clock the hour that matches a routine activity.
2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	EE.2.MD.8. Recognize that money has value.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and interpret data.	
2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	EE.2.MD.9-10. Create picture graphs from collected measurement data.
2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	

Second Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[8] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	EE.2.G.1. Identify common two-dimensional shapes: square, circle, triangle, and rectangle.
2.G.2. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.	Not applicable.
2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as *two halves*, *three thirds*, *four fourths*. Recognize that equal shares of identical wholes need not have the same shape.	Not applicable. See EE.4.G.3 and EE.4.NF.1–2.

[8] Sizes are compared directly or visually, not compared by measuring.

Third Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and solve problems involving multiplication and division.	
3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	EE.3.OA.1-2. Use repeated addition to find the total number of objects and determine the sum.
3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Not applicable See EE.3.OA.1 and EE.5.NBT.5.
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*	EE.3.OA.4. Solve addition and subtraction problems when result is unknown, limited to operands and results within 20.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand properties of multiplication and the relationship between multiplication and division.	
3.OA.5. Apply properties of operations as strategies to multiply and divide.[9] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Not applicable. See EE.N-CN.2.
3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	Not applicable. See EE.5.NBT.6–7.
CLUSTER: Multiply and divide within 100.	
3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Not applicable. See EE.7.NS.2.a and EE.7.NS.2.b.
CLUSTER: Solve problems involving the four operations, and identify and explain patterns in arithmetic.	
3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[10]	EE.3.OA.8. Solve one-step real-world problems using addition or subtraction within 20.
3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	EE.3.OA.9. Identify arithmetic patterns.

[9] Students need not use formal terms for these properties.

[10] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.

Third Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use place value understanding and properties of operations to perform multi-digit arithmetic.[11]	
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.	EE.3.NBT.1. Use decade numbers (10, 20, 30) as benchmarks to demonstrate understanding of place value for numbers 0–30.
3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	EE.3.NBT.2. Demonstrate understanding of place value to tens.
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	EE.3.NBT.3. Count by tens using models such as objects, base ten blocks, or money.

[11] A range of algorithms may be used.

Third Grade Mathematics Domain: Number and Operations—Fractions[12]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Develop understanding of fractions as numbers.	
3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b.*	EE.3.NF.1–3. Differentiate a fractional part from a whole.
3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.	
3.NF.2.a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.	
3.NF.2.b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	
3.NF.3.a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	
3.NF.3.b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	
3.NF.3.c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	
3.NF.3.d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

[12] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

Third Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	
3.MD.1. Tell and write time to the nearest minute, and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	EE.3.MD.1. Tell time to the hour on a digital clock.
3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[13] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[14]	EE.3.MD.2. Identify the appropriate measurement tool to solve one-step word problems involving mass and volume.
CLUSTER: Represent and interpret data.	
3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	EE.3.MD.3. Use picture or bar graph data to answer questions about data.
3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	EE.3.MD.4. Measure length of objects using standard tools, such as rulers, yardsticks, and meter sticks.

[13] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[14] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Geometric measurement: understand concepts of area, and relate area to multiplication and to addition.	
3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.	Not applicable. See EE.4.MD.2.
3.MD.5.a. A square with side length of 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	
3.MD.5.b. A plane figure, which can be covered without gaps or overlaps by *n* unit squares, is said to have an area of *n* square units.	
3.MD.6. Measure areas by counting unit squares (square cm, square m, square in., square ft, and improvised units).	
3.MD.7. Relate area to the operations of multiplication and addition.	
3.MD.7.a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	
3.MD.7.b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	
3.MD.7.c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.	
3.MD.7.d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	
CLUSTER: Geometric measurement: recognize perimeter as an attribute of plane figures, and distinguish between linear and area measures.	
3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Not applicable. See EE.7.G.4 and EE.8.G.9.

Third Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason with shapes and their attributes.	
3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	EE.3.G.1. Describe attributes of two-dimensional shapes.
3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	EE.3.G.2. Recognize that shapes can be partitioned into equal areas.

Fourth Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use the four operations with whole numbers to solve problems.	
4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	EE.4.OA.1-2. Demonstrate the connection between repeated addition and multiplication.
4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	EE.4.OA.3. Solve one-step real-world problems using addition or subtraction within 100.
CLUSTER: Gain familiarity with factors and multiples.	
4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	EE.4.OA.4. Show one way to arrive at a product.
CLUSTER: Generate and analyze patterns.	
4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	EE.4.OA.5. Use repeating patterns to make predictions.

Fourth Grade Mathematics Domain: Numbers and Operations in Base Ten[15]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Generalize place value understanding for multi-digit whole numbers.	
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	Not applicable. See EE.5.NBT.1.
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	EE.4.NBT.2. Compare whole numbers to 10 using symbols (<, >, =).
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.	EE.4.NBT.3. Round any whole number 0-30 to the nearest ten.
CLUSTER: Use place value understanding and properties of operations to perform multi-digit arithmetic.	
4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.	EE.4.NBT.4. Add and subtract two-digit whole numbers.
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable. See EE.4.OA.1.
4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Not applicable.

[15] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Fourth Grade Mathematics Domain: Number and Operations—Fractions[16]

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend understanding of fraction equivalence and ordering.	
4.NF.1. Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	EE.4.NF.1–2. Identify models of one half (1/2) and one fourth (1/4).
4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
CLUSTER: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	
4.NF.3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.	EE.4.NF.3. Differentiate between whole and half.
4.NF.3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	
4.NF.3.b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	
4.NF.3.c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	
4.NF.3.d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	

[16] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

CCSS Grade-Level Standards	DLM Essential Elements
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Not applicable. See EE.4.OA.1–2 and EE.5.NBT.5.
4.NF.4.a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*	
4.NF.4.b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	
4.NF.4.c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	
CLUSTER: Understand decimal notation for fractions, and compare decimal fractions.	
4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[17] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	Not applicable. See EE.7.NS.2.c-d.
4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	
4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	

[17] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Fourth Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	
4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*	EE.4.MD.1. Identify the smaller measurement unit that comprises a larger unit within a measurement system (inches/foot, centimeter/meter, minutes/hour).
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	EE.4.MD.2.a. Tell time using a digital clock. Tell time to the nearest hour using an analog clock.
	EE.4.MD.2.b. Measure mass or volume using standard tools.
	EE.4.MD.2.c. Use standard measurement to compare lengths of objects.
	EE.4.MD.2.d. Identify coins (penny, nickel, dime, quarter) and their values.
4.MD.3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.*	EE.4.MD.3. Determine the area of a square or rectangle by counting units of measure (unit squares).
CLUSTER: Represent and interpret data.	
4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	EE.4.MD.4.a. Represent data on a picture or bar graph given a model and a graph to complete.
	EE.4.MD.4.b. Interpret data from a picture or bar graph.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Geometric measurement: understand concepts of angle and measure angles.	
4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:	EE.4.MD.5. Recognize angles in geometric shapes.
4.MD.5.a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	
4.MD.5.b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	EE.4.MD.6. Identify angles as larger and smaller.
4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Not applicable. See EE.4.G.2.a.

Fourth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	
4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	EE.4.G.1. Recognize parallel lines and intersecting lines.
4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	EE.4.G.2. Describe the defining attributes of two-dimensional shapes.
4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures, and draw lines of symmetry.	EE.4.G.3. Recognize that lines of symmetry partition shapes into equal areas.

Fifth Grade Mathematics Domain: Operations and Algebraic Thinking

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Write and interpret numerical expressions.	
5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Not applicable.
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Not applicable.
CLUSTER: Analyze patterns and relationships.	
5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	EE.5.OA.3. Identify and extend numerical patterns.

Fifth Grade Mathematics Domain: Number and Operations in Base Ten

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand the place value system.	
5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	EE.5.NBT.1. Compare numbers up to 99 using base ten models.
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	EE.5.NBT.2. Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than.
5.NBT.3. Read, write, and compare decimals to thousandths.	EE.5.NBT.3. Compare whole numbers up to 100 using symbols (<, >, =).
5.NBT.3.a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).	
5.NBT.3.b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
5.NBT.4. Use place value understanding to round decimals to any place.	EE.5.NBT.4. Round two-digit whole numbers to the nearest 10 from 0—90.
CLUSTER: Perform operations with multi-digit whole numbers and with decimals to hundredths.	
5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm.	EE.5.NBT.5. Multiply whole numbers up to 5 × 5.
5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	EE.5.NBT.6–7. Illustrate the concept of division using fair and equal shares.
5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	

Fifth Grade Mathematics Domain: Number and Operations—Fractions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use equivalent fractions as a strategy to add and subtract fractions.	
5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	EE.5.NF.1. Identify models of halves (1/2, 2/2) and fourths (1/4, 2/4, 3/4, 4/4).
5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	EE.5.NF.2. Identify models of thirds (1/3. 2/3, 3/3) and tenths (1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 10/10).
CLUSTER: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
5.NF.3. Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Not applicable. See EE.6.RP.1.
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	Not applicable.
5.NF.4.a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*	

CCSS Grade-Level Standards	DLM Essential Elements
5.NF.4.b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
5.NF.5. Interpret multiplication as scaling (resizing), by:	Not applicable.
5.NF.5.a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	
5.NF.5.b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Not applicable. See EE.10.N-CN.2.b.
5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[18]	Not applicable. See EE.7.NS.2.b.
5.NF.7.a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*	
5.NF.7.b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*	

[18] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

CCSS Grade-Level Standards	DLM Essential Elements
5.NF.7.c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	

Fifth Grade Mathematics Domain: Measurement and Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Convert like measurement units within a given measurement system.	
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.	EE.5.MD.1.a. Tell time using an analog or digital clock to the half or quarter hour.
	EE.5.MD.1.b. Use standard units to measure weight and length of objects.
	EE.5.MD.1.c. Indicate relative value of collections of coins.
CLUSTER: Represent and interpret data.	
5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	EE.5.MD.2. Represent and interpret data on a picture, line plot, or bar graph.
CLUSTER: Geometric measurement: understand concepts of volume, and relate volume to multiplication and to addition.	
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	EE.5.MD.3. Identify common three-dimensional shapes.
5.MD.3.a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.	
5.MD.3.b. A solid figure, which can be packed without gaps or overlaps using *n* unit cubes, is said to have a volume of *n* cubic units.	

CCSS Grade-Level Standards	DLM Essential Elements
5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft, and improvised units.	EE.5.MD.4–5. Determine the volume of a rectangular prism by counting units of measure (unit cubes).
5.MD.5. Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume.	
5.MD.5.a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	
5.MD.5.b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.	
5.MD.5.c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	

Fifth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Graph points on the coordinate plane to solve real-world and mathematical problems.	
5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	EE.5.G.1-4. Sort two-dimensional figures and identify the attributes (angles, number of sides, corners, color) they have in common.
5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	
CLUSTER: Classify two-dimensional figures into categories based on their properties.	
5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	EE.5.G.1-4. Sort two-dimensional figures and identify the attributes (angles, number of sides, corners, color) they have in common.
5.G.4. Classify two-dimensional figures in a hierarchy based on properties.	

Sixth Grade Mathematics Domain: Ratios and Proportional Relationships

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand ratio concepts, and use ratio reasoning to solve problems.	
6.RP.1. Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	EE.6.RP.1. Demonstrate a simple ratio relationship.
6.RP.2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[19]	Not applicable. See EE.7.RP.1–3.
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	Not applicable. See EE.8.F.1–3.
6.RP.3.a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	
6.RP.3.b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	
6.RP.3.c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	

[19] Expectations for unit rates in this grade are limited to non-complex fractions.

CCSS Grade-Level Standards	DLM Essential Elements
6.RP.3.d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

Sixth Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4), and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	EE.6.NS.1. Compare the relationships between two unit fractions.
CLUSTER: Compute fluently with multi-digit numbers, and find common factors and multiples.	
6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.	EE.6.NS.2. Apply the concept of fair share and equal shares to divide.
6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	EE.6.NS.3. Solve two-factor multiplication problems with products up to 50 using concrete objects and/or a calculator.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of numbers to the system of rational numbers.	
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	EE.6.NS.5–8. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	
6.NS.6.a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.	
6.NS.6.b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	
6.NS.6.c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
6.NS.7. Understand ordering and absolute value of rational numbers.	
6.NS.7.a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*	
6.NS.7.b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3° C > –7° C to express the fact that –3°C is warmer than –7°C.*	
6.NS.7.c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*	
6.NS.7.d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	

CCSS Grade-Level Standards	DLM Essential Elements
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	EE.6.NS.5–8. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

Sixth Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of arithmetic to algebraic expressions.	
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	EE.6.EE.1–2. Identify equivalent number sentences.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.	
6.EE.2.a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from 5" as 5 –* y*.*	
6.EE.2.b. Identify parts of an expression using mathematical terms (*sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	
6.EE.2.c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$*.*	
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 +* x*) to produce the equivalent expression 6 + 3*x*; apply the distributive property to the expression 24*x *+ 18*y *to produce the equivalent expression 6(4*x *+ 3*y*); apply properties of operations to* y + y + y *to produce the equivalent expression 3*y*.*	EE.6.EE.3. Apply the properties of addition to identify equivalent numerical expressions.
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* y + y + y *and 3*y *are equivalent because they name the same number regardless of which number* y *stands for.*	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason about and solve one-variable equations and inequalities.	
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	EE.6.EE.5–7. Match an equation to a real-world problem in which variables are used to represent numbers.
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Not applicable.
CLUSTER: Represent and analyze quantitative relationships between dependent and independent variables.	
6.EE.9.Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	Not applicable.

Sixth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve real-world and mathematical problems involving area, surface area, and volume.	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	EE.6.G.1. Solve real-world and mathematical problems about area using unit squares.
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	EE.6.G.2. Solve real-world and mathematical problems about volume using unit cubes.
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Not applicable.
6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Not applicable.

Sixth Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Develop understanding of statistical variability.	
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	EE.6.SP.1–2. Display data on a graph or table that shows variability in the data.
6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Not applicable. See EE.S-ID.4.
CLUSTER: Summarize and describe distributions.	
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Not applicable. See EE.6.SP.1–2.
6.SP.5. Summarize numerical data sets in relation to their context, such as by:	EE.6.SP.5. Summarize data distributions shown in graphs or tables.
6.SP.5.a. Reporting the number of observations.	
6.SP.5.b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	
6.SP.5.c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	
6.SP.5.d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

Seventh Grade Mathematics Domain: Ratios and Proportional Relationships

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Analyze proportional relationships and use them to solve real-world and mathematical problems.	
7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*	EE.7.RP.1–3. Use a ratio to model or describe a relationship.
7.RP.2. Recognize and represent proportional relationships between quantities.	
7.RP.2.a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	
7.RP.2.b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	
7.RP.2.c. Represent proportional relationships by equations. *For example, if total cost* t *is proportional to the number* n *of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as* t = pn*.*	
7.RP.2.d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	
7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

Seventh Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	
7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	EE.7.NS.1. Add fractions with like denominators (halves, thirds, fourths, and tenths) with sums less than or equal to one.
7.NS.1.a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	
7.NS.1.b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	
7.NS.1.c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	
7.NS.1.d. Apply properties of operations as strategies to add and subtract rational numbers.	
7.NS.2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	See below.
7.NS.2.a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	EE.7.NS.2.a. Solve multiplication problems with products to 100.
7.NS.2.b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.	EE.7.NS.2.b. Solve division problems with divisors up to five and also with a divisor of 10 without remainders.
7.NS.2.c. Apply properties of operations as strategies to multiply and divide rational numbers	EE.7.NS.2.c–d. Express a fraction with a denominator of 10 as a decimal.
7.NS.2.d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	

CCSS Grade-Level Standards	DLM Essential Elements
7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers.[20]	EE.7.NS.3. Compare quantities represented as decimals in real-world examples to tenths.

[20] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Seventh Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use properties of operations to generate equivalent expressions.	
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	EE.7.EE.1. Use the properties of operations as strategies to demonstrate that expressions are equivalent.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* a + *0.05*a = *1.05*a *means that "increase by 5%" is the same as "multiply by 1.05."*	EE.7.EE.2. Identify an arithmetic sequence of whole numbers with a whole number common difference.
CLUSTER: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	Not applicable.
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	EE.7.EE.4. Use the concept of equality with models to solve one-step addition and subtraction equations.
7.EE.4.a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	

CCSS Grade-Level Standards	DLM Essential Elements
7.EE.4.b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	

Seventh Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Draw, construct, and describe geometrical figures and describe the relationships between them	
7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	EE.7.G.1. Match two similar geometric shapes that are proportional in size and in the same orientation.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	EE.7.G.2. Recognize geometric shapes with given conditions.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	EE.7.G.3. Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
CLUSTER: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	
7.G.4. Know the formulas for the area and circumference of a circle, and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	EE.7.G4. Determine the perimeter of a rectangle by adding the measures of the sides.
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	EE.7.G.5. Recognize angles that are acute, obtuse, and right.
7.G.6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	EE.7.G.6. Determine the area of a rectangle using the formula for length × width, and confirm the result using tiling or partitioning into unit squares.

Seventh Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use random sampling to draw inferences about a population.	
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	EE.7.SP.1–2. Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	
CLUSTER: Draw informal comparative inferences about two populations.	
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	EE.7.SP.3. Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.
7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	Not applicable. See EE.S-ID.4.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Investigate chance processes, and develop, use, and evaluate probability models.	
7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	EE.7.SP.5–7. Describe the probability of events occurring as possible or impossible.
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	
7.SP.7.a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*	
7.SP.7.b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	

CCSS Grade-Level Standards	DLM Essential Elements
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	Not applicable.
7.SP.8.a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	
7.SP.8.b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	
7.SP.8.c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

Eighth Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Know that there are numbers that are not rational, and approximate them by rational numbers.	
8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	EE.8.NS.1. Subtract fractions with like denominators (halves, thirds, fourths, and tenths) with minuends less than or equal to one.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	EE.8.NS.2.a. Express a fraction with a denominator of 100 as a decimal.
	EE.8.NS.2.b. Compare quantities represented as decimals in real-world examples to hundredths.

Eighth Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Work with radicals and integer exponents.	
8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	EE.8.EE.1. Identify the meaning of an exponent (limited to exponents of 2 and 3).
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	EE.8.EE.2. Identify a geometric sequence of whole numbers with a whole number common ratio.
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	EE.8.EE.3–4. Compose and decompose whole numbers up to 999.
8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation, and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
CLUSTER: Understand the connections between proportional relationships, lines, and linear equations.	
8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	EE.8.EE.5–6. Graph a simple ratio by connecting the origin to a point representing the ratio in the form of *y*/*x*. *For example, when given a ratio in standard form (2:1), convert to 2/1, and plot the point (1,2).*
8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Analyze and solve linear equations and pairs of simultaneous linear equations.	
8.EE.7. Solve linear equations in one variable.	EE.8.EE.7. Solve simple algebraic equations with one variable using addition and subtraction.
8.EE.7.a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).	
8.EE.7.b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
8.EE.8. Analyze and solve pairs of simultaneous linear equations.	Not applicable. See EE.8.EE.5–6.
8.EE.8.a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	
8.EE.8.b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3*x *+ 2*y *= 5 and 3*x *+ 2*y *= 6 have no solution because 3*x *+ 2*y *cannot simultaneously be 5 and 6.*	
8.EE.8.c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Eighth Grade Mathematics Domain: Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Define, evaluate, and compare functions.	
8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[21]	EE.8.F.1–3. Given a function table containing at least 2 complete ordered pairs, identify a missing number that completes another ordered pair (limited to linear functions).
8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
CLUSTER: Use functions to model relationships between quantities.	
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	EE.8.F.4. Determine the values or rule of a function using a graph or a table.
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	EE.8.F.5. Describe how a graph represents a relationship between two quantities.

[21] Function notation is not required in Grade 8.

Eighth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand congruence and similarity using physical models, transparencies, or geometry software.	
8.G.1. Verify experimentally the properties of rotations, reflections, and translations:	EE.8.G.1. Recognize translations, rotations, and reflections of shapes.
8.G.1.a. Lines are taken to lines, and line segments to line segments of the same length.	
8.G.1.b. Angles are taken to angles of the same measure.	
8.G.1.c. Parallel lines are taken to parallel lines.	
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	EE.8.G.2. Identify shapes that are congruent.
8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Not applicable.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	EE.8.G.4. Identify similar shapes with and without rotation.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	EE.8.G.5. Compare any angle to a right angle, and describe the angle as greater than, less than, or congruent to a right angle.
CLUSTER: Understand and apply the Pythagorean Theorem.	
8.G.6. Explain a proof of the Pythagorean Theorem and its converse.	Not applicable.
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Not applicable.
CLUSTER: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	
8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.	EE.8.G.9. Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and volume of rectangular prisms).

Eighth Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Investigate patterns of association in bivariate data.	
8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Not applicable.
8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Not applicable. See EE.10.S-ID.1–2 and EE.10.S-ID.3.
8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Not applicable.
8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	EE.8.SP.4. Construct a graph or table from given categorical data, and compare data categorized in the graph or table.

High School Mathematics Domain: Number and Quantity—The Real Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the properties of exponents to rational exponents.	
N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*	EE.N-RN.1. Determine the value of a quantity that is squared or cubed.
N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Not applicable.
CLUSTER: Use properties of rational and irrational numbers.	
N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Not applicable.

High School Mathematics Domain: Number and Quantity—Quantities★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason quantitatively, and use units to solve problems.	
N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	EE.N-Q.1–3. Express quantities to the appropriate precision of measurement.
N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.	
N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform arithmetic operations with complex numbers.	
N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.	Not applicable.
N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	EE.N-CN.2.a. Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
	EE.N-CN.2.b. Solve real-world problems involving addition and subtraction of decimals, using models when needed.
	EE.N-CN.2.c. Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.
N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Not applicable.
CLUSTER: Represent complex numbers and their operations on the complex plane.	
N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Not applicable.
N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*	Not applicable.
N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Use complex numbers in polynomial identities and equations.	
N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	Not applicable.
N-CN.8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	Not applicable.
N-CN.9. (+) Know the Fundamental Theorum of Algebra; show that it is true for quadratic polynomials	Not applicable.

High School Mathematics Domain: Number and Quantity – Vector and Matrix Quantities

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Represent and model with vector quantities.	
N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $\|v\|$, $\|\|v\|\|$, v).	Not applicable.
N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Not applicable.
N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	Not applicable.
CLUSTER: Perform operations on vectors.	
N-VM.4. (+) Add and subtract vectors.	Not applicable.
N-VM.4.a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	
N-VM.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	
N-VM.4.c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
N-VM.5. (+) Multiply a vector by a scalar.	Not applicable.
N-VM.5.a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	
N-VM.5.b. Compute the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform operations on matrices, and use matrices in applications.	
N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	Not applicable.
N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Not applicable.
N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	Not applicable.
N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Not applicable.
N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	Not applicable.
N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Not applicable.
N-VM.12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Not applicable.

High School Mathematics Domain: Algebra—Seeing Structure in Expressions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Interpret the structure of expressions.	
A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★	EE.A-SSE.1. Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.
A-SSE.1.a. Interpret parts of an expression, such as terms, factors, and coefficients.	
A-SSE.1.b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.	
A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	Not applicable.
CLUSTER: Write expressions in equivalent forms to solve problems.	
A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★	EE.A-SSE.3. Solve simple algebraic equations with one variable using multiplication and division.
A-SSE.3.a. Factor a quadratic expression to reveal the zeros of the function it defines.	
A-SSE.3.b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
A-SSE.3.c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	
A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	EE.A-SSE.4. Determine the successive term in a geometric sequence given the common ratio.

High School Mathematics Domain: Algebra—Arithmetic with Polynomials and Rational Expressions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Perform arithmetic operations on polynomials.	
A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Not applicable.
CLUSTER: Understand the relationship between zeros and factors of polynomials.	
A-APR.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	Not applicable.
A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Not applicable.
CLUSTER: Use polynomial identities to solve problems.	
A-APR.4. Prove polynomial identities, and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	Not applicable.
A-APR.5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[22]	Not applicable.

[22] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Rewrite rational expressions.	
A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	Not applicable.
A.APR.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	Not applicable.

High School Mathematics Domain: Algebra—Creating Equations★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Create equations that describe numbers or relationships.	
A-CED.1. Create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	EE.A-CED.1. Create an equation involving one operation with one variable, and use it to solve a real-world problem.
A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	EE.A-CED.2–4. Solve one-step inequalities.
A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	
A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* V = IR *to highlight resistance* R.	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand solving equations as a process of reasoning, and explain the reasoning.	
A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Not applicable.
A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Not applicable. See EE.A-CED.1.
CLUSTER: Solve equations and inequalities in one variable.	
A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Not applicable. See EE.A-CED.1.
A-REI.4. Solve quadratic equations in one variable.	Not applicable.
A-REI.4.a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	
A-REI.4.b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b.	
CLUSTER: Solve systems of equations.	
A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Not applicable.
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	Not applicable. See EE.A-REI.10–12.

CCSS Grade-Level Standards	DLM Essential Elements
A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	Not applicable. See EE.A-REI.10–12.
A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	Not applicable.
A-REI.9. (+) Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Not applicable.
CLUSTER: Represent and solve equations and inequalities graphically.	
A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	EE.A-REI.10–12. Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*
A-REI.11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	
A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

High School Mathematics Domain: Functions—Interpreting Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand the concept of a function, and use function notation.	
F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	EE.F-IF.1–3. Use the concept of function to solve problems.
F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* f*(0)* = f*(1)* = *1,* f(n + *1)* = f(n) + f(n – *1) for* n ≥ *1.*	
CLUSTER: Interpret functions that arise in applications in terms of the context.	
F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★	EE.F-IF.4–6. Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.
F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n*) gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★	
F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Analyze functions using different representations.	
F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	Not applicable. See EE.F-IF.1–3.
F-IF.7.a. Graph linear and quadratic functions, and show intercepts, maxima, and minima.	
F-IF.7.b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.7.c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	
F-IF.7.d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
F-IF.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Not applicable.
F-IF.8.a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.8.b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	
F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	Not applicable.

High School Mathematics Domain: Functions—Building Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Build a function that models a relationship between two quantities.	
F-BF.1. Write a function that describes a relationship between two quantities.★	EE.F-BF.1. Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.
F-BF.1.a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.1.b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	
F-BF.1.c. (+) Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather ballon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*	Not applicable.
F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	EE.F-BF.2. Determine an arithmetic sequence with whole numbers when provided a recursive rule.
CLUSTER: Build new functions from existing functions.	
F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases, and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	Not applicable.
F-BF.4. Find inverse functions.	Not applicable.
F-BF.4.a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	
F-BF.4.b. (+) Verify by composition that one function is the inverse of another.	
F-BF.4.c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	

CCSS Grade-Level Standards	DLM Essential Elements
F-BF.4.d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	
F-BF.5. (+) Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	Not applicable.

High School Mathematics Domain: Functions—Linear, Quadratic, and Exponential Models★

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Construct and compare linear, quadratic, and exponential models, and solve problems.	
F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	EE.F-LE.1–3. Model a simple linear function such as $y = mx$ to show that these functions increase by equal amounts over equal intervals.
F-LE.1.a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	
F-LE.1.b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
F-LE.1.c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	Not applicable.
CLUSTER: Interpret expressions for functions in terms of the situation they model.	
F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.	Not applicable. See EE.F-IF.1–3.

High School Mathematics Domain: Functions—Trigonometric Functions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Extend the domain of trigonometric functions using the unit circle.	
F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Not applicable.
F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Not applicable.
F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of since, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	Not applicable.
F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Not applicable.
CLUSTER: Model periodic phenomena with trigonometric functions.	
F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	Not applicable.
F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Not applicable.
F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology; and interpret them in terms of the context.*	Not applicable.
CLUSTER: Prove and apply trigonometric identities	
F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	Not applicable.
F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Experiment with transformations in the plane.	
G.CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	EE.G-CO.1. Know the attributes of perpendicular lines, parallel lines, and line segments; angles; and circles.
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Not applicable.
G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Not applicable.
G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	EE.G-CO.4–5. Given a geometric figure and a rotation, reflection, or translation of that figure, identify the components of the two figures that are congruent.
G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
CLUSTER: Understand congruence in terms of rigid motions.	
G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	EE.G-CO.6–8. Identify corresponding congruent and similar parts of shapes.
G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Prove geometric theorems.	
G-CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	Not applicable.
G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	Not applicable.
G-CO.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	Not applicable.
CLUSTER: Make geometric constructions.	
G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	Not applicable.
G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand similarity in terms of similarity transformations.	
G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:	Not applicable. See EE.G-CO.6–8.
G-SRT.1.a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	
G-SRT.1.b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Not applicable. See EE.G-CO.6–8.
G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Not applicable. See EE.G-CO.6–8.
CLUSTER: Prove theorems involving similarity.	
G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	Not applicable.
G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Not applicable. See EE.G-CO.6–8.
CLUSTER: Define trigonometric ratios, and solve problems involving right triangles.	
G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Not applicable.
G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	Not applicable.
CLUSTER: Apply trigonometry to general triangles.	
G-SRT.9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Not applicable.
G-SRT.10. (+) Prove the Laws of Sines and Cosines, and use them to solve problems.	Not applicable.
G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	Not applicable.

High School Mathematics Domain: Geometry—Circles

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and apply theorems about circles.	
G-C.1. Prove that all circles are similar.	Not applicable.
G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Not applicable.
G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Not applicable.
G-C.4. (+) Construct a tangent line from a point outside a give circle to the circle.	Not applicable.
CLUSTER: Find arc lengths and areas of sectors of circles.	
G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Not applicable.

High School Mathematics Domain: Geometry—Expressing Geometric Properties with Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Translate between the geometric description and the equation for a conic section.	
G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Not applicable.
G-GPE.2. Derive the equation of a parabola given a focus and directrix.	Not applicable.
G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Not applicable.
CLUSTER: Use coordinates to prove simple geometric theorems algebraically.	
G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	Not applicable.
G-GPE.5. Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Not applicable. See EE.G.CO.1.
G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Not applicable. See EE.G.CO.1.
G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	EE.G-GPE.7. Find perimeters and areas of squares and rectangles to solve real-world problems.

High School Mathematics Domain: Geometry—Geometric Measurement and Dimension

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Explain volume formulas, and use them to solve problems.	
G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	EE.G-GMD.1–3. Make a prediction about the volume of a container, the area of a figure, and the perimeter of a figure, and then test the prediction using formulas or models.
G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	Not applicable.
G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	Not applicable. See EE.8.G.9 and EE.G-GPE.7.
CLUSTER: Visualize relationships between two-dimensional and three-dimensional objects.	
G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	EE.G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects.

High School Mathematics Domain: Geometry—Modeling with Geometry

CLUSTER: Apply geometric concepts in modeling situations.	
G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	EE.G-MG.1–3. Use properties of geometric shapes to describe real-life objects.
G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	
G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	

High School Mathematics Domain: Statistics and Probability★—Interpreting Categorical and Quantitative Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Summarize, represent, and interpret data on a single count or measurement variable.	
S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	EE.S-ID.1–2. Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	EE.S-ID.3. Interpret general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	EE.S-ID.4. Calculate the mean of a given data set (limit the number of data points to fewer than five).
CLUSTER: Summarize, represent, and interpret data on two categorical and quantitative variables.	
S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Not applicable. See EE.F-IF.1 and EE.A-REI.6–7.
S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Not applicable.
S-ID.6.a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	
S-ID.6.b. Informally assess the fit of a function by plotting and analyzing residuals.	
S-ID.6.c. Fit a linear function for a scatter plot that suggests a linear association.	
CLUSTER: Interpret linear models.	
S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Not applicable. See EE.F-IF.4–6.

CCSS Grade-Level Standards	DLM Essential Elements
S-ID.8. Compute (using technology), and interpret the correlation coefficient of a linear fit.	Not applicable.
S-ID.9. Distinguish between correlation and causation.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Making Inferences and Justifying Conclusions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and evaluate random processes underlying statistical experiments.	
S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	EE.S-IC.1–2. Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	
CLUSTER: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	
S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Not applicable. See EE.S-ID.1–2.
S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Not applicable. See EE.S-ID.1–2.
S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Not applicable. See EE.S-ID.1–2.
S-IC.6. Evaluate reports based on data.	Not applicable. See EE.S-ID.1–2.

High School Mathematics Domain: Statistics and Probability—Conditional Probability and the Rules of Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand independence and conditional probability, and use them to interpret data.	
S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	EE.S-CP.1–5. Identify when events are independent or dependent.
S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	
S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	
CLUSTER: Use the rules of probability to compute probabilities of compound events in a uniform probability model.	
S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	Not applicable. See EE.S-IC.1–2.
S-CP.7. Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.	Not applicable. See EE.S-IC.1–2.

CCSS Grade-Level Standards	DLM Essential Elements
S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.	Not applicable.
S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Using Probability to Make Decisions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Calculate expected values, and use them to solve problems.	
S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	Not applicable.
S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	Not applicable.
S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*	Not applicable.
S-MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	Not applicable.
CLUSTER: Use probability to evaluate outcomes of decisions.	
S-MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	Not applicable.
S-MD.5.a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	
S-MD.5.b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	
S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Not applicable.

Source: Mississippi Code 37-1-3, 37-16-3