MISSISSIPPI DEPARTMENT OF EDUCATION

Ensuring a bright future for every child

2014 Mississippi College- and Career-Readiness Standards for Mathematics

COMMON CORE STATE STANDARDS INITIATIVE

PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Effective Date: 2014-2015 School Year

Mississippi College- and Career- Readiness Standards for Mathematics

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\+ Course includes content from the *2007 Mississippi Mathematics Framework Revised*
\# Course includes content from the *2007 Mississippi Mathematics Framework Revised* and the *Common Core State Standards for Mathematics*

INTRODUCTION

Mission Statement

The Mississippi Department of Education is dedicated to student success including the improvement of student achievement in mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. The *2014 Mississippi College- and Career- Readiness Standards for Mathematics* ("The Standards") provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level and course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy.

Purpose

In an effort to closely align instruction for students who are progressing toward postsecondary study and the workforce, the *2014 Mississippi College- and Career-Readiness Standards for Mathematics* include grade- and course-specific standards for K-12 mathematics.

The primary purpose of this document is to provide a basis for curriculum development for K-12 mathematics teachers, outlining what students should know and be able to do by the end of each grade level and course. Courses for grades K-12 are based on the *Common Core State Standards for Mathematics* and *Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards for Mathematics* (www.corestandards.org). These courses also include the *PARCC Model Content Frameworks* (www.parcconline.org/parcc-model-content-frameworks) to support implementation of the standards and assessments. Mississippi-specific courses that were revised to align with the *Common Core State Standards for Mathematics* include Pre-Calculus (renamed Algebra III) and Calculus.

The Southern Regional Education Board (SREB) Math Ready course is included as a transition to college mathematics course.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Implementation

The required year for the *2014 Mississippi College- and Career- Readiness Standards for Mathematics* is school year 2014-2015.

Technology

The Mississippi Department of Education (MDE) strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The appropriate use of instructional technology is integrated throughout the *2014 Mississippi College- and Career- Readiness Standards for Mathematics.* Teaching strategies at each grade level and in every secondary course incorporate technology in the form of calculators, software, or on-line internet resources. The graphing calculator is an integral part of mathematics courses beginning with Grade 6.

The MDE believes strongly in the Principles and Standards for School Mathematics Technology Principle of the National Council for Teachers of Mathematics (NCTM):

> "Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. In the mathematics classrooms envisioned in *Principles and Standards*, every student has access to technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking."
> (NCTM, 2013, http://www.nctm.org.)

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COMMITTEE MEMBERS (2007)

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Common Core State Standards for Mathematics Overview

COMMON CORE STATE STANDARDS FOR MATHEMATICS OVERVIEW

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge. Aiming for clarity and specificity, these Standards endeavor to follow a design that not only stresses conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific expectations but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary for college and/or careers. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with reading disabilities the use of Braille, screen reader technology, or other assistive devices should be made available. In addition, while writing, these students should have access to a scribe, computer, or speech-to-text technology in their classroom. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of College- and Career- Readiness for all students.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the 2014 Mississippi College- and Career- Readiness Standards for Mathematics

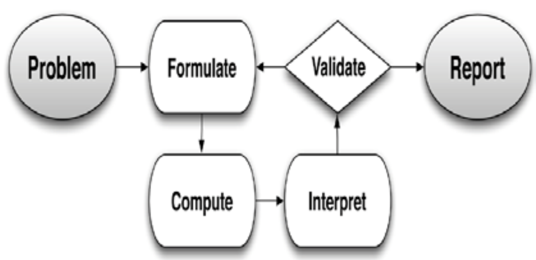
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to the Standards in mathematics instruction.

The Standards are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Modeling (High School Courses only)

Modeling standards are noted throughout the high school courses with an asterisk (*). *Modeling* links classroom mathematics and statistics to everyday life, work, and decision-making. *Modeling* is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.



Making mathematical models is a Standard for Mathematical Practice, and specific *Modeling* standards appear throughout the high school standards. The basic modeling cycle above involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

Partnership for Assessment of Readiness of College and Careers (PARCC)

Overview of PARCC

The Partnership for Assessment of Readiness for College and Careers (PARCC) is a consortium of 19 states plus the District of Columbia and the U.S. Virgin Islands working together to develop a common set of K-12 assessments in English and math anchored in what it takes to be ready for college and careers. These new K-12 assessments will build a pathway to College- and Career- Readiness by the end of high school, mark students' progress toward this goal from 3rd grade up, and provide teachers with timely information to inform instruction and provide student support. Mississippi is a governing member of PARCC and maintains a variety of positions within the PARCC infrastructure.

The PARCC Vision

PARCC states have committed to building a K-12 assessment system that:

- Builds a pathway to College- and Career- Readiness for all students,
- Creates high-quality assessments that measure the full range of the Common Core State Standards,
- Supports educators in the classroom,
- Makes better use of technology in assessments, and
- Advances accountability at all levels.

The PARCC Assessments

PARCC represents a fundamental shift in how we think about testing, state to state. PARCC is based on the core belief that assessment should not be a process to penalize educators and districts, but rather a tool for enhancing teaching and learning. The PARCC assessments in mathematics are carefully crafted to give teachers, schools, students and parents better and more useful information on how they are preparing students for careers and college — and life. Educators, specialists, and administrators within each PARCC state voluntarily work(ed) together to develop a common set of K-12 assessments in mathematics.

PARCC is committed to developing tests worth taking and are the type of new generation assessment that teachers have requested for years. In math, students will have to solve complex problems, show their work, and demonstrate how they solved the problem. Unlike pencil-and-paper bubble tests, these new assessments will more closely resemble high-quality classroom work. PARCC will measure what children are learning, in a more meaningful way.

PARCC's estimated testing time is reasonable and reflects the shift from traditional multiple choice tests to performance-based assessments. The new assessments are designed to measure the full range of knowledge and skills students need to be career and college ready or on track toward that goal, through performance based tasks. The assessments will be innovative in design and more engaging for students.

PARCC assessments will measure the full range of student performance, including the performance of high- and low-achieving students. As a governing member of PARCC, Mississippi plays a key role in all implementation and policy decisions regarding the PARCC assessments.

Purpose of the PARCC Model Content Frameworks

Included in the Introduction page of each grade/course are key features of the PARCC Model Content Frameworks, Grades K-12. As part of its proposal to the U.S. Department of Education, PARCC committed to developing model content frameworks for mathematics to serve as a bridge between the Common Core State Standards and the PARCC assessments. PARCC developed the PARCC Model Content Frameworks to: (1) inform development of item specifications and blueprints for the PARCC assessments, and (2) support implementation of the *Common Core State Standards for Mathematics.*

The PARCC Model Content Frameworks were developed through a state-led process that included mathematics content experts in PARCC member states and members of the Common Core State Standards writing team. Although the primary purpose of the Model Content Frameworks is to provide a frame for the PARCC assessments, they also are voluntary resources to help educators and those developing curricula and instructional materials.

Guidance from the PARCC Model Content Frameworks is written with the expectation that students develop content knowledge, conceptual understanding and expertise with the Standards for Mathematical Practice. The Introduction of each grade level or course standards in the *2014 Mississippi College- and Career- Readiness Standards for Mathematics* includes the following *Examples* from the PARCC Model Content Frameworks include:

- *Examples of key advances from the previous grade;*
- *Fluency expectations or examples of culminating standards;*
- *Examples of major within-grade dependencies;*
- *Examples of opportunities for connections among standards, clusters or domains;*
- *Examples of opportunities for in-depth focus;*
- *Examples of opportunities for connecting mathematical content and mathematical practices;*
- *Content emphases by cluster; and*
- *Fluency recommendations*

Connections to the PARCC Assessment

The PARCC Assessment System will be designed to measure the knowledge, skills, and understandings essential to achieving college and career readiness. In mathematics, this includes conceptual understanding, procedural skill and fluency, and application and problem solving, as defined by the standards. Each of these works in conjunction with the others to promote students' achievement in mathematics. To measure the full range of the standards, the assessments will include tasks that require students to connect mathematical content and mathematical practices.

The Model Content Frameworks for Mathematics reflect these priorities by providing detailed information about selected practice standards, fluencies, connections, and content emphases. These emphases will be reflected in the PARCC Assessment System.

The Model Content Frameworks do not contain a suggested scope and sequence by quarter. Rather, they provide examples of key content dependencies (where one concept ought to come before another), key instructional emphases, opportunities for in-depth work on key concepts, and connections to critical practices. These last two components, in particular, intend to support local and state efforts to deliver instruction that connects content and practices while achieving the standards' balance of conceptual understanding, procedural skill and fluency, and application.

Overall, the PARCC Assessment System will include a mix of items, including short- and extended-response items, performance-based tasks, and technology-enhanced items. In mathematics, the items will be designed to elicit evidence of whether students can:

- Solve problems involving the Major work of the grade with connections to the practice standards;
- Solve problems involving the Additional and Supporting work of the grade with connections to the practice standards;
- Express mathematical reasoning by constructional mathematical arguments and critiques;
- Solve real-world problems by engaging particularly in the modeling practice; and
- Demonstrate fluency in the areas set for in the content standards for Grades 3–6.

Questions asked will measure student learning within and across various mathematical domains and practices. The questions will cover the full range of mathematics, including conceptual understanding, procedural fluency, and the varieties of expertise described by the practice standards. Mathematical understanding, procedural skill, and the ability to apply what one knows are equally important and can be assessed using mathematical tasks of sufficient richness, which PARCC will include in its assessment system.

It is critical that all students are able to demonstrate mastery of the skills and knowledge described in the standards. PARCC recognizes the importance of equity, access and fairness in its assessments and aligned materials.

PARCC Assessment Emphases

Each tested grade level or course describes cluster emphases for content standards. These are provided because curriculum, instruction, and assessment in each course must reflect the focus and emphasis of the grade level/course standards. To make relative emphases in the Standards more transparent and useful, cluster headings are designated as **Major**, **Additional** and **Supporting**. Key: ■ Major Clusters; ◘ or ■ Supporting Clusters; and ⭘ or ■ Additional Clusters.

Some clusters that are not major emphases in themselves are designed to *support* and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade would fairly be called *additional*. To say that some things have greater emphasis is not to say that anything in the Standards can be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

All standards figure in a mathematical education and therefore will be *eligible* for inclusion on the PARCC end-of-year assessments. The assessments will mirror the message that is communicated here: Major clusters will be a majority of the assessment, Supporting clusters will be assessed through their success at supporting the Major clusters and Additional clusters will be assessed as well. The assessments will strongly focus where the Standards strongly focus.

Finally, the following are some recommendations for using the cluster-level emphases:

Do …

- Use the guidance to inform instructional decisions regarding time and other resources spent on clusters of varying degrees of emphasis.

- Allow the focus on the major work of the grade to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.

- Evaluate instructional materials taking the cluster-level emphases into account. The major work of the grade must be presented with the highest possible quality; the supporting work of the grade should indeed support the major focus, not detract from it.

- Set priorities for other implementation efforts taking the emphases into account, such as staff development; new curriculum development; or revision of existing formative or summative testing at the district or school level.

Don't …

- Neglect any material in the standards. (Instead, use the information provided to connect Supporting Clusters to the other work of the grade.)

- Sort clusters from Major to Supporting, and then teach them in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.

- Use the cluster headings as a replacement for the standards. All features of the standards matter — from the practices to surrounding text to the particular wording of individual content standards. Guidance is given at the cluster level as a way to talk about the content with the necessary specificity yet without going so far into detail as to compromise the coherence of the standards.

Reading the College- and Career- Readiness Standards

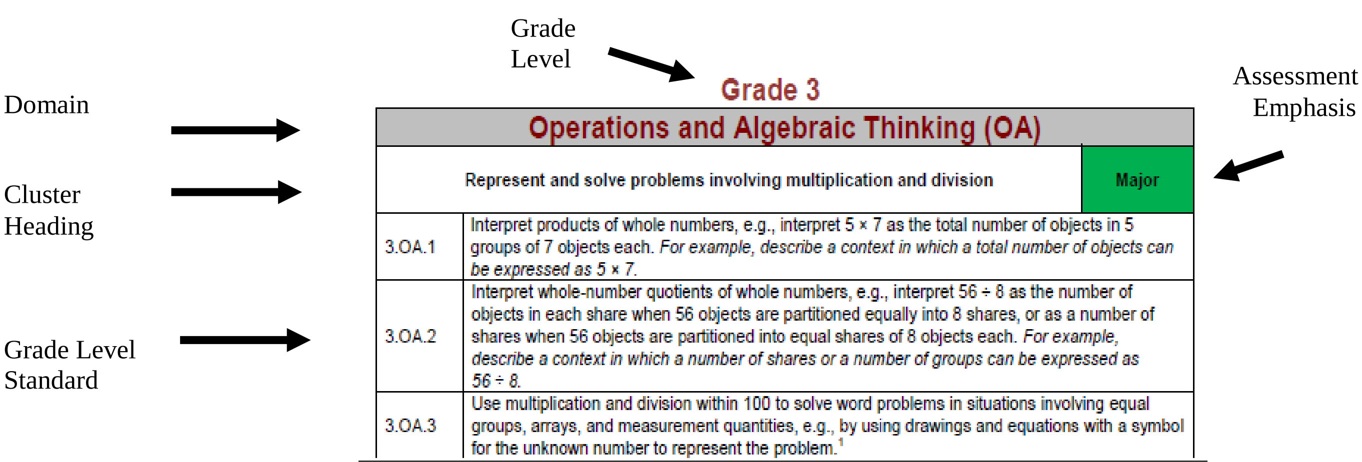
- Standards define what students should understand and be able to do.

- Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

- Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

- Conceptual Categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

- Assessment Emphasis is included. Major clusters will be a majority of the assessment, supporting clusters will be assessed through their success at supporting the major clusters and additional clusters will be assessed as well.



Course Title

Conceptual Category
(HS courses only)

Domain

Cluster Heading

Course Standard

Assessment Emphasis

Integrated Mathematics I

Geometry		
Congruence (G-CO)		
Experiment with transformations in the plane		**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	

College- and Career- Readiness Standards for Mathematics (Grades K-5)

Mathematics | Grade K

In Kindergarten, instruction should focus on two critical areas: (1) representing, relating, and operating on whole numbers- initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics. Each critical area is described below.

> (1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

> (2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics |Grade K

PARCC Model Content Frameworks Indications

Examples of Key Advances in Kindergarten

Students with academically oriented families and/or preschool experience may enter Kindergarten able to recite number words and use counting to answer "how many?" questions. But not all children have these background experiences. Fewer still will know what addition and subtraction do, let alone finding sums and differences. Kindergarten establishes these and other foundations and begins the process of building the mathematical habits of mind that lead to proficient mathematical practice and are described in the Standards for Mathematical Practice.

- Students learn to pair objects 1-1 with counting words, and they learn that the last number word tells the number of objects in a collection (up to 20). This is called "cardinal counting," as opposed to "rote counting" (merely being able to recite the counting words in order).
- Some students will progress from the "counting all" strategy to the more sophisticated strategy of "counting on" during Kindergarten (see K.CC.B.4c and 1.OA.C.5).
- Students learn to compare the number of objects in one group versus another group, and eventually to compare written numerals 1–10.
- Students understand addition as joining collections and adding to collections, and they understand subtraction as taking collections apart or taking from collections, representing these operations in a variety of ways.

Fluency Expectations or Examples of Culminating Standards

K.CC.A.3 This standard refers to written numerals from 0–20. In particular, students should work throughout the year toward fluency in writing the numerals 0–10.[1]

K.CC.B.5 This standard refers to cardinal counting and producing a collection with a given number of objects. Students should become fluent in cardinal counting well before the end of Kindergarten, because much else in Kindergarten depends on it.

[1] "Learning to write number symbols (numerals) is a much more difficult task than is reading them and often is not begun until kindergarten. Writing numerals requires children to have an accurate mental image of the symbol, which entails left-right orientation, and a motor plan to translate the mental image into the correct sequence of motor actions to form a numeral…. Some numerals are much easier than others. The loops in 6 and 9, the curve and straight line in the 2, and the crossovers in the 8 are difficult but can be mastered by kindergarten children with effort. The easier numerals 1, 3, 4, 5, and 7 can often be mastered earlier. Whenever children do learn to write numerals, learning to write correct and readable numerals is not enough. They must become fluent at writing numerals (i.e., writing numerals must become over-learned) so that writing them as part of a more complex task is not so slow or effortful as to be discouraging when solving several problems. It is common for children at this step and even later to reverse some numerals (such as 3) because the left-right orientation is difficult for them. This will become easier with age and experience." *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council (2009), p. 138.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

K.CC.C.7 Compare two numbers 1–10 presented as written numerals.

K.OA.A.5 Fluently add and subtract within 5. That is, given any two numbers 0–5 with sum less than or equal to 5, students can name the sum reasonably quickly; and likewise for related differences, given one number and a goal that is 5 or less, they can reasonably quickly name the "missing amount."

Examples of Major Within-Grade Dependencies

Standards listed later in a given domain may often refer to skills that depend on standards listed earlier in the domain. However, there are many connections between skills, within and across domains. (See some Opportunities for Connections below.) Regardless of the logical ordering of the standards, *teaching* need not (and should not) tick off standards one by one in order. For example, the "later standards" may suggest interesting situations that call for counting skills that the children are still developing, and that motivate that development. And many fluencies and skills must typically develop steadily throughout the year, with opportunities for practice and links to developing conceptual understanding.

- Much of the learning in Kindergarten—K.CC.C.6, all of K.OA and K.NBT, and K.MD.B.3—depends on the foundational ability to count to answer "how many?" (K.CC.5), which, itself, is grounded in K.CC.B.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Working with numbers 11–19 (K.NBT) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3).
- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.A.3). See below under "Examples of Linking Supporting Clusters to the Major Work of the Grade."
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers; see below under "Examples of Linking Supporting Clusters to the Major Work of the Grade."

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

K.CC.B.5 Cardinal counting should be a focus in itself as needed, and should be a main component of other work in the Kindergarten classroom. Opportunities to develop students' understanding of cardinality

abound, both within the instructional time devoted specifically to mathematics (as noted above) and elsewhere in the instructional day.

K.OA.A.2 Through representing and solving addition and subtraction problems (see also K.OA.A.1), students understand addition as joining and adding to, and understand subtraction as separating and taking from. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time, with subtraction coming to be understood as reversing the actions involved in addition and as finding an unknown addend (see 1.OA.B.4).[2]

K.OA.A.3 Connected with other standards such as K.OA.A.1 and K.OA.A.2, the ability to decompose numbers flexibly is a key focus. At this age, even children who seem competent at counting can hold conversations like this:

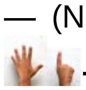
—Happy birthday, Naomi! Tell the class how old you are today.

—I'm *six!* (Naomi shows .)

—Oh, so you are *this* old? (Teacher shows .)

— (Naomi giggles and says.) *No!* I'm *this!* (And she re- shows .)

—Oh, so are you *this* old, right? (Teacher shows .)

— (Naomi giggles again.) *Noooo!* I'm *this old!* (And she again shows .)

Naomi can count, mostly accurately, but seemed to believe there was only one correct way to show 6 and so didn't pursue the matter further!

K.OA.A. 4 **"**Making ten" will become a key strategy (in grade 1) for adding and subtracting within 20; students gain the foundations for this in Kindergarten by finding the number that makes 10 when given another number. Over the course of the year, given playful opportunities (e.g., a "how many fingers *don't* you see" game), many Kindergarten children can become fluent with the pairs of numbers that make 10, and can, when a

[2] pp. 2, 3 of the *Progression* document for K Counting and Cardinality and K-5 Operations and Algebraic Thinking. Available at www.ime.arizona.edu/progressions.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

number less than 10 is named, name the “missing amount” even without looking at fingers.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected to the of the grade. Class discussions and mathematical tasks (short, longer, scaffolded and unscaffolded) are an important opportunity to develop good practice while learning new content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the standards for mathematical practice follow.

Kindergarten students *speak* the number names by ones and by tens all the way to 100 (K.CC.A.1). The structure of a number name like “thirty-two” reflects the underlying system of place value. Attending to and using that structure (MP.7) is, even more than K.NBT.1, an important foundation for place value. Children who can recite “thirty, forty, fifty, sixty,…” can also hear the rhyme in “thirty-eight, forty-eight, fifty-eight, sixty-eight,…” counting by 10s not starting at 0. Hearing this structure prepares them to read those numbers in grade 1. See the *Progression* document for K Counting and Cardinality and K-5 Operations and Algebraic Thinking for more information about how patterns in the number names affect learning (including deviations from those patterns, as in “sixteen” which puts the ones digit first).

- As children learn to count by tens (K.CC.A.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence ten, twenty, thirty… as a new child joins the ones already standing in front of the classroom and showing all their fingers. Connecting *six* children with the spoken *sixty*, and *seven* children with the spoken *seventy*, and *eight* children with the spoken *eighty* also draws attention to structure (MP.7) in a way that prepares for place value.

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Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). The equations that the teacher writes on the board to describe these situations (such as 8 + 2 = 10) are also mathematical models.
- If a student chooses to use objects, fingers, or a math drawing to analyze and solve a word problem, then it is an example of the student using an appropriate tool strategically (MP.5).

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[3] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

[3] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◻ Supporting Clusters; ⬡ Additional Clusters

Counting and Cardinality

■ A. Know number names and the count sequence.
■ B. Count to tell the number of objects.
■ C. Compare numbers

Operations and Algebraic Thinking

■ A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

■ A. Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

⬡ A. Describe and compare measurable attributes.
◻ B. Classify objects and count the number of objects in categories.

Geometry

⬡ A. Identify and describe shapes.
◻ B. Analyze, compare, create, and compose shapes.

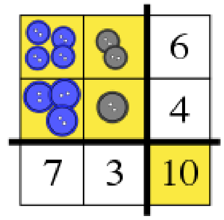
Examples of Linking Supporting Clusters to the Major Work of the Grade

So much is brand new to children in Kindergarten that as much as possible, everything throughout the school day should support everything else, as for example when language supports number.

- Even within mathematics itself, understanding for example that 18 is “ten ones and [eight more] ones” (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose 10 in more than one way (see K.OA.A.3). For example, given a collection of 10 buttons, children could classify by color and size to answer (K.CC.B.5) questions like “how many *small* buttons do you have” or “how many blue buttons do you have” or “how many large gray buttons do you have?” Such a decomposition of objects can show both 10 = 7 + 3 and 10 = 6 + 4. (See figure.)

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)



- Students can count vertices (see K.CC.B.5) as a strategy for recognizing shapes in different orientations (see K.G.A.2) and can use shapes as a setting in which to compare numbers (see K.CC.C.6; e.g., count to see which has more vertices, an octagon or a hexagon—see K.G.B.4).

Grade K

Counting and Cardinality (CC)

Know number names and the count sequence		**Major**
K.CC.1	Count to 100 by ones and by tens.	
K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	
K.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	
Count to tell the number of objects		**Major**
K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	
K.CC.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	
Compare numbers		**Major**
K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]	
K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.	

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from		**Major**
K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	
K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	
K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	
K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	
K.OA.5	Fluently add and subtract within 5.	

Grade K

Number and Operations in Base Ten (NBT)		
Work with numbers 11-19 to gain foundations for place value		**Major**
K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	
Measurement and Data (MD)		
Describe and compare measurable attributes		**Additional**
K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	
K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	
Classify objects and count the number of objects in each category		**Supporting**
K.MD.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]	
Geometry (G)		
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)		**Additional**
K.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind*, and *next to.*	
K.G.2	Correctly name shapes regardless of their orientations or overall size.	
K.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
Analyze, compare, create, and compose shapes		**Supporting**
K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	
K.G.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	
K.G.6	Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	

[1] Include groups with up to ten objects.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

[3] Limit category counts to be less than or equal to 10.

Grade K

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 1

In Grade 1, instruction should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. Each critical area is described below.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations,**

Grade 1

Geometry, Statistics & Probability (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

[1] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade K to Grade 1

- Grade K students determined sums and differences primarily by representing problems in concrete terms. In grade 1, students gradually come to use more sophisticated strategies (such as making ten) that depend on the properties of addition and subtraction.
- Given the larger numbers studied in grade 1, students must begin to use and understand the symbol system for writing numbers. This includes reading and writing numbers through 120, and understanding the early elements of place value, in particular being able to think of a ten as a unit and understanding that the digits of a two-digit number represent the number of complete tens in that number, and the number of remaining ones.
- Using this understanding of place value and the properties of operations, grade 1 students will represent, explain, and perform addition and subtraction of two-digit numbers in specified cases.
- Students in K learned about the meanings of addition and subtraction as ways of finding what happens when collections are combined or separated, or when their sizes are changed. Building on this base of meaning, grade 1 students represent and solve a large variety of addition and subtraction problems (word problems and problems set in classroom discussions of various scenarios involving putting together, taking apart, comparing, and so on, with different quantities in the problem unknown).[4]

Grade K students generally saw equations only when the teacher wrote them on the board; Kindergarten students were not expected to write equations themselves. Grade 1 students will write equations for a variety of reasons, such as expressing a decomposition of a number (16 = 9 + 7), expressing a piece of reasoning about numbers (9 + 7 = 9 + 1 + 6 along the way to making ten), or representing a word problem with an unknown (9 + ? = 16). Students use the equal sign appropriately, evaluate the truth of an equation, and determine unknown numbers that will make an equation true.

Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Students are fluent in all additions and subtractions within 10. (Note that with sufficient experiences and practice throughout the year, many students will also become fluent with additional additions and subtractions beyond 10, e.g., 10 + 6 = 16, or 17 + 1 = 18.)

[4] See Table 2, p. 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

1.NBT.C.5 Students are fluent at mentally finding 10 more or 10 less than any given two-digit number without having to count. (This standard is best thought of as an indicator of whether students are understanding place value for two-digit numbers.)

The explicit fluency standard 1.OA.C.6 also has implications for fluency in other standards, including:

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. This standard relates to fluency when the additions and subtractions are within 10, as they are in the italicized examples given along with the standard: *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. This standard is closely related to fact families; for example, knowing the fact family for 8 + 3 = 11 means being able to find the unknown number in 8 + ? = 11.

Examples of Major Within-Grade Dependencies

- Standard 1.OA.B.3 calls for students to "apply properties of operations" and gives the example "If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known." Similarly, knowing 13 – 3 gives a good starting place for figuring out 13 – 4. Use of properties lets us apply knowledge we have to situations we need to figure out. But *all* of that depends on having some starting places. Standards 1.OA.C.6 and 1.NBT.C.5 are such starting places, and are essential building blocks for all of the arithmetic of grade 1. They must therefore be given ample attention early in the year. Though often notated on paper, these, and 1.NBT.6, are essentially *mental* arithmetic knowledge and reasoning (though 1.NBTB..6 is still treated as developing, not yet fluent and fully mental).

- 1.NBT.B.2 describes the place-value foundations for 1.NBT.B., and 1.NBT.C.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- A thorough understanding of how place value language and notation (which are mostly consistent with each other in English) represents number (1.NBT.A) serves calculation (1.NBT.B) in many ways—not just pencil and paper calculation, but mental calculation as well. It is valuable for calculation to know that numbers are

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

named so that "twenty-eight" with the "eight" taken away leaves "twenty." That is, the names are designed to make that calculation easy so that we can base harder calculations like 28 – 9 and 28 – 7 on it using properties of the operations 1.OA.3. Similarly "twenty-eight" with the "twenty" taken away leaves "eight," allowing us to base harder calculations on *that*.

The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multi-digit addition and subtraction (1.NBT). For example, when teaching a new situation type such as Compare, the numbers can initially be small enough so that a math drawing shows all of the objects in the problem; this keeps the focus on the situation, and allows students not yet fluent to absorb it. Once the situation type is better understood, and students grow in fluency, larger numbers can begin to appear in such problems. At this point an equation can be used to represent the problem, and fluency and computational skill become involved in finding the answer.[5]

- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, Put Together/Take Apart problems with Addend Unknown can show subtraction as finding an unknown addend.[6]

- Units are a connection between place value (1.NBT) and measurement (1.MD). Grade 1 is when students first encounter the concept of a tens unit, and it is also when they first encounter the concept of a length unit. In later grades, unit thinking will become important throughout arithmetic, including in the development of multi-digit multiplication and division algorithms, and the development of fraction concepts and operations.

- Measurement standards 1.MD.A.1 and 1.MD.A.2, together, support and provide a context for the 1.OA.1 goal of solving subtraction problems that involve comparing. To meet standard 1.MD.A.1, students compare the lengths of two objects by means of a third object. In some cases, that third object might be a length of string that allows a

[5] See p. 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.
[6] See p. 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

"copy" of the length of one immovable object to be moved to another location to compare with the length of another movable object. But that third object might be some kind of unit—the difference between two students' heights measured in inch cubes; or the size of two whales, measured in students stretched end-to-end. When students cannot find the *exact* difference because of the magnitude of the numbers that arise from measurement—as may occur in comparing two students' heights—they may still compare the measurements to know which is greater (1.NBT.3). (Grade 2 standard 2.MD.6 formalizes this idea on a number line diagram.) While children are dealing only with the limited precision of whole and half-hours, they must distinguish the position of the longer hand (the "big hand"). From the top of the circle at the top of the hour (not a common expression in American English) pointing to 12, that hand must go *half* way around the circle over the course of half of an hour. Then another half way around for another half hour—the two halves making a full hour's time. The fact that half way around takes them from 12 to the number that is half of 12 may *not* be a good connection to make—the half hour is not 6 of any standard units, nor is the hour 12 of them—but geometry standard 1.G.A.3, partitioning circles into halves and quarters, does connect well.

- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This is also connected to length measurement (1.MD.A.2), since students must visualize an object to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two 5s into a 10 uses, the idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations is a big idea, common across mathematics.

Examples of Opportunities for In-Depth Focus

1.NBT.B.2 Grade 1 is students' first encounter with the three linked components of the place value system: base ten units, bundling and unbundling of units, and positional notation. Understanding place value is the foundation of the entire NBT domain.

1.NBT.C.4 Understanding place value is not a final goal on its own; the goal is to use place value understanding and properties of operations to add and

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PARCC Model Content Frameworks Indications (continued)

subtract (1.NBT.B). Students learn how standard notation presents and/or records problems for which students have developed mental strategies—adding 10 (1.NBTC..5) repeatedly and adding 1 repeatedly (counting on, 1.OA.C.6) to a two-digit number—and extends that to adding two arbitrary two-digit numbers (with a result within 100). Being able to represent these additions with materials that show the base 10 structure, having a strong mental image and ability with simple versions of these additions, and understanding how the notation records these additions, and being able to interpret subtraction in its relation to addition are the foundation for all future arithmetic in elementary school.

1.OA.A.1 There are many distinct elementary addition and subtraction situations; students in grade 1 should work extensively with all of them. (The harder situation types need not be mastered until grade 2.)[7]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected to the of the grade. Class discussions and mathematical tasks (short, longer, scaffolded and unscaffolded) are an important opportunity to develop good practice while learning new content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the standards for mathematical practice follow.

- All work with properties (1.OA.B.3) and with understanding and using place value (e.g., 1.NBT.B.2, 1.NBT.C.4) should be seen as an investigation and use of the structure of our number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts (1.NBT.C.4, 1.NBT.C.5, 1.NBT.C.6) are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief, but excellent arguments at this grade level could include:
    - I know that 7 – 3 equals 4 because 4 + 3 equals 7 (shows 1.OA.4 being met).
    - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
    - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.

[7] See Table 2, p. 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

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PARCC Model Content Frameworks Indications (continued)

- The experience of starting at some number (e.g., 23) and *counting* on 10, and then 10 more, and then 10 more, and then 10 more, and so on, and *hearing* the repetition "thirty-three, forty-three, fifty-three, sixty-three) is often a bit of a surprise to children, and quite powerful. From the repeated reasoning, they abstract a pattern (MP.8): they describe this
  in various ways, but sometimes say variations on "adding 10 to any number rhymes" and it "changes the counting-by-tens word I use." Thus, content standard 1.NBT.4 is being approached by applying standard MP.8 of mathematical practice.

- Students in grade 1 work with some sophisticated addition and subtraction situations (1.OA.1), such as "Julie has 8 more apples than Lucy. Julie has 12 apples. How many apples does Lucy have?" The equations 12 – 8 = ? and ? + 8 = 12 are both mathematical models of this situation (MP.4).

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[8] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[8] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

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PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	A. Represent and solve problems involving addition and subtraction
■	B. Understand and apply properties of operations and the relationship between addition and subtraction.
■	C. Add and subtract within 20.
■	D. Work with addition and subtraction equations.
Number and Operations in Base Ten	
■	A. Extend the counting sequence.
■	B. Understand place value.
■	C. Use place value understanding and properties of operations to add and subtract.
Measurement and Data	
■	A. Measure lengths indirectly and by iterating length units.
○	B. Tell and write time.
□	C. Represent and interpret data.
Geometry	
○	A. Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

When students work with organizing, representing, and interpreting data, the process include practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.4 after the semicolon, and see the K-5 MD Progression document, especially Table 1 on page 4 and the discussion of categorical data on pp. 5, 6).[9]

Telling and writing time on digital clocks (1.MD.3) is a context in which one can practice reading numbers (1.NBT.1), a kind of "application," but no more. Relating those times to *meanings*—events during a day—is not part of the 1.MD.3 *content* standard, but making sense of what one is doing (MP.1), and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

[9] http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

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Operations and Algebraic Thinking (OA)		
Represent and solve problems involving addition and subtraction		**Major**
1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]	
1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	
Understand and apply properties of operations and the relationship between addition and subtraction		**Major**
1.OA.3	Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	
1.OA.4	Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	
Add and subtract within 20		**Major**
1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	
1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	
Work with addition and subtraction equations		**Major**
1.OA.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	
1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*	

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Number and Operations in Base Ten (NBT)

Extend the counting sequence		**Major**
1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	
Understand place value		**Major**
1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	
Use place value understanding and properties of operations to add and subtract		**Major**
1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	
1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	
1.NBT.6	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	

Measurement and Data (MD)

Measure lengths indirectly and by iterating length units		**Major**
1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	
1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps*.	

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Tell and write time		**Additional**
1.MD.3	Tell and write time in hours and half-hours using analog and digital clocks.	
Represent and interpret data		**Supporting**
1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	
Geometry (G)		
Reason with shapes and their attributes		**Additional**
1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	
1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]	
1.G.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	

[2] See Glossary, Table 1.

[3] Students need not use formal terms for these properties.

[4] Students do not need to learn formal names such as “right rectangular prism.”

Grade 1

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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In Grade 2, instruction should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Each critical area is described below.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 2

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 1 to Grade 2

- Where Grade 1 students worked within 100, Grade 2 students will read and write numbers through 1000, extending their understanding of place value to include units of hundreds.
- Similarly, Grade 2 students use their understanding of place value to add and subtract within 1000 (e.g., 237 + 616 or 822 – 237). They can explain what they are doing as they add and subtract, and they become fluent in the special case of addition and subtraction within 100.
- For situation-based problems (e.g., word problems), students extend their ability by solving two-step problems using addition, subtraction, or both operations. They also master the harder kinds of one-step addition and subtraction problems in this grade (such as Take From with Start Unknown).[10]
- Students use standard units of measure and appropriate measurement tools. They understand basic properties of linear (length/distance) measurement, such as the fact that the smaller the unit, the more iterations will be needed to cover a given length.

Fluency Expectations or Examples of Culminating Standards

2.OA.B.2 Fluently add and subtract within 20 mentally. By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (Students can also show their fluency using an efficient, general algorithm.)[11]

Note: The following standards don't explicitly state a fluency requirement, but fluency is also important in these cases for the reasons stated.

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, 100s. (A lack of fluency here can signal a lack of understanding. Skip counting is also sometimes a strategy for adding or subtracting, so fluency is helpful.)

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (Students who struggle to read a three-digit number may not grasp place value.)

2.NBT.B.8 Given any number between 100 and 900, mentally add or subtract 10 or 100. (A lack of fluency here can signal a lack of understanding.)

[10] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

[11] For the difference between a computation *strategy* and a computation *algorithm*, see the Glossary, page 330).

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PARCC Model Content Frameworks Indications (continued)

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (Sufficient practice is required in order to measure accurately and reasonably quickly.)

Students' ability to *explain* (2.NBT.B.9) what they know in a fluent and clear way will continue to develop for years, but the *understanding* reflected in that standard should be a major focus begun relatively early in grade 2 and should be considered a "culminating standard" at the end of the year, an understanding that can be *assumed* in later grades.

Examples of Major Within-Grade Dependencies

- Cluster 2.NBT.A—understanding place value—is the foundation for cluster 2.NBT.B, using place value understanding and the properties of operations to add and subtract. (This does not mean that 2.NBT.A must be entirely "completed" before beginning work with 2.NBT.B; mastery of the two clusters can grow over time in tandem with one another.)
- Knowing single-digit sums from memory (2.OA.B.2) is required for adding and subtracting multi-digit numbers fluently and efficiently (2.NBT.B).
- Cluster 2.NBT.B in turn is an essential foundation for *all* the major arithmetic work of grade 2 and beyond.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Representing whole numbers as lengths (2.MD.B.6) and comparing measurements (2.MD.A.4) can build a robust and flexible model for fluent subtraction (2.OA.A.1). For example, a good way to see the "distance" from 6 to 20 is to see the distance from 6 to 10 joined with the distance from 10 to 20. (See *Complements* in "Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice," below.)
- Problems involving dollars, dimes, and pennies (2.MD.C.8) should be connected with the place value learning of 100s, 10s, and 1s (2.NBT.A.1). Though the notation is different, a dollar is 100 cents, a "bundle" of ten dimes, each of which is a "bundle" of ten pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes), and additions that are appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is *not* a standard at this grade).

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PARCC Model Content Frameworks Indications (continued)

- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multi-digit addition and subtraction (2.OA.B.2, 2.NBT.B).[12]

- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with groups of 5 (2.OA.C.4). (See also *Knowing 5* in "Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice," below.) Recognizing time by seeing the minute hand at 3 and *knowing* that is fifteen minutes; recognizing three nickels as 15¢; and seeing the 15-ness of a 3-by-5 rectangular array held in any position at all (including with neither base horizontal) will prepare for understanding what the new operation of multiplication *means*, and not just the particular facts that it uses.

Examples of Opportunities for In-Depth Focus

2.OA.A.1 Using situations (from word problems, from classroom events or experiences, and from discovered mathematical patterns) as a source of problems can help students make sense of and contextualize the operations they are learning. There is a tremendous variety of basic situation types in addition and subtraction.[13]

2.NBT.B.7 It takes substantial time throughout the year for students to extend addition and subtraction to 1000, connecting steps in the computation to what they know about place value and properties of operations.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Fidelity to mathematical practice should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to develop good practice while learning new

[12] Easier word problem types, such as Result Unknown, should generally involve more difficult sums and differences. The challenge in these problems is not algebraic, but rather computational, and the problems can build computational skill in context. Harder situation types, such as Start Unknown, or Compare, should sometimes involve easy computations and sometimes involve harder computations. (Having harder computations with a harder situation type may be counterintuitive, but if the harder situation types always involve easier numbers, then students will never have need or incentive to represent them with equations, when means they won't have opportunities to connect the structure of the representation to the structure of the problem.)

[13] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf as well as www.achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems

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PARCC Model Content Frameworks Indications (continued)

content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow. In all grades, there is plentiful opportunity for curriculum and instruction to meet the content standards with fidelity to mathematical practice. Standard MP.1 should be part of the life of the classroom at all grades and at most times. The standards for using and expressing the regularity in repeated reasoning (MP.8) and looking for and making use of structure (MP.7) are especially well suited to two major content areas of Grade 2, and might be the principal aspects of practice to take advantage of at this grade level.

- *Complements:* Given the opportunity, children fairly readily generalize the idea of partners of 10, and they feel proud of being able to name pairs, say, to 30 (e.g., knowing that 12 and 18 are ‘partners of 30’). After practice pairing only multiples of 10, they can name pairs to 100. This approach—learning a set of “content” facts and recognizing/building a structure behind them (MP.7) through repeated use in varied contexts (MP.8)—makes very appropriate use of good mathematical practice.
- *Doubling and Halving:* Seeing even numbers as the sum of two equal addends (“doubling”) and seeing the number of pairs contained within an even number (“halving”) is an extremely powerful foundation for multiplication and its central property (the distributive property) and for later study of fractions. If doubling is not restricted just to numbers less than 20, but extended to include two-digit numbers with digits less than 5, students can use physical and, not long after, *mental* images of dimes and pennies, or base ten rods and units, to double numbers like “four hundred” to “eight hundred” (spoken) easily as they double 4 to 8, and later to double 40 to 80 the same way, and still later hold enough in their heads to double 32 and 24 (with no more mathematical knowledge, but more increase in working memory and executive function to track the process). Likewise, they can halve two-digit numbers both of whose digits are even by picturing these as composed of dimes and pennies (or some other material that distinguishes 10s and 1s). The *practice* is just with basic facts that are part of grade 2 standards, and the varied contexts helps make these secure and also prepare the ground for later learning, so that the distributive property of multiplication in Grade 3 is not a brand new idea but has already become a familiar structure (MP.7) with an experience-based (MP.8) and intuitive underpinning. In the same way, halving becomes a robust model for other fractions.

Although children in Grades K-1 have used objects and drawings as appropriate tools to represent mathematical ideas and operations, Grade 2 is the first time that students make more than passing use of objects that are not just school-artifacts, like blocks and

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PARCC Model Content Frameworks Indications (continued)

rods, but remain appropriate tools (MP.5) for a lifetime: rulers, clocks, coins, and the number line (essentially an abstract ruler or measurement scale).

- *Knowing 5:* Standard MP.5 is about not just the ability to *use* tools, but the ability to *choose* the appropriate tool for a task. At this stage, because students are just beginning to use a variety of tools, their utility may seem both obvious and fixed to a task: a ruler measures length; a clock measures time; a coin "measures" money, and children have little to make choices about. But coming to understand the *significance* of counting by 5—the usefulness of that litany and the situations in which it appears—may involve, for some children, a choice of which of several images (nickels, hands, telling time) is most clarifying and salient to them. Generating the abstraction—the litany 0, 5, 10, 15, etc.—may also be aided by experiences in the various domains, the recognition that one sequence of numbers is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[14] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.[15]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[14] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[15] Reference unknown.

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

■ A. Represent and solve problems involving addition and subtraction
■ B. Add and subtract within 20.
□ C. Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

■ A. Understand place value.
■ B. Use place value understanding and properties of operations to add and subtract.

Measurement and Data

■ A. Measure and estimate lengths in standard units.
■ B. Relate addition and subtraction to length.
□ C. Work with time and money.
□ D. Represent and interpret data.

Geometry

○ A. Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with time and money (2.MD.C), their work with dollars, dimes, and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.2), and with arrays of five rows and/or five columns (2.OA.C) should be taken together.
- In cluster 2.MD.D, "Represent and interpret data," it is particularly standard 2.MD.10 that represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can be a visually appealing context for solving addition and subtraction problems. The language in 2.MD.10 mentions word problems (2.OA) explicitly. See the *Progression* document for K-5 Measurement and Data for more on the connections between data work and arithmetic in the early grades.[16]

[16] http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

- Without adding greater meaning or depth, 2.MD.9 is a potential context for 2.MD.1 and gives students a first taste of visual comparison of numerical information (though the fact that this numerical information was derived from length makes the representation more about scaling the information than about visualizing it).

Grade 2

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction		**Major**
2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	
Add and subtract within 20		**Major**
2.OA.2	Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.	
Work with equal groups of objects to gain foundations for multiplication		**Supporting**
2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	

Number and Operations in Base Ten (NBT)

Understand place value		**Major**
2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	
2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.	
2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	
2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	
Use place value understanding and properties of operations to add and subtract		**Major**
2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	
2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	

Grade 2

2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

Measurement and Data (MD)

Measure and estimate lengths in standard units	**Major**
2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.
2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Relate addition and subtraction to length	**Major**
2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
Work with time and money	**Supporting**
2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
Represent and interpret data	**Supporting**
2.MD.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Grade 2

Geometry (G)		
Reason with shapes and their attributes		Additional
2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	
2.G.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	

[1] See Glossary, Table 1.

[2] See standard 1.OA.6 for a list of mental strategies.

[3] Explanations may be supported by drawings or objects.

[4] See Glossary, Table 1.

[5] Sizes are compared directly or visually, not compared by measuring.

Grade 2

Additional Resources:

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 3

In Grade 3, instruction should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Each critical area is described below.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 2 to Grade 3

- Students in grade 3 begin to enlarge their concept of number by developing an understanding of fractions as numbers. This work will continue in grades 3-6, preparing the way for work with the complete rational number system in grades 6 and 7.

- Students in grades K-2 worked on number; place value; and addition and subtraction concepts, skills and problem solving. Beginning in grade 3, students will learn concepts, skills, and problem solving for multiplication and division. This work will continue in grades 3, 4 and 5, preparing the way for work with ratios and proportions in grades 6 and 7.

Fluency Expectations or Examples of Culminating Standards

- **3.OA.C.7** Students fluently multiply and divide within 100. By the end of grade 3, they know all products of two one-digit numbers from memory.

- **3.NBT.A.2** Students fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (Although **3.OA.C.7** and **3.NBT.A.2** are both fluency standards, these two standards do not represent equal investments of time in grade 3. Note that students in grade 2 were already adding and subtracting within 1000, just not fluently. That makes **3.NBT.2** a relatively small and incremental expectation. By contrast, multiplication and division are new in grade 3, and meeting the multiplication and division fluency standard **3.OA.C.7** with understanding is a major portion of students' work in grade 3.)

Examples of Major Within-Grade Dependencies

- Students must begin work with multiplication and division (**3.OA**) at or near the very start of the year to allow time for understanding and fluency to develop. Note that area models for products are an important part of this process (**3.MD.C.7**). Hence, work on concepts of area (**3.MD.C.5–6**) should likely begin at or near the start of the year as well.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with partitioning shapes (**3.G.A.2**) relates to visual fraction models (**3.NF**)**.**

- Scaled picture graphs and scaled bar graphs (**3.MD.B.3**) can be a visually appealing context for solving multiplication and division problems.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

3.OA.A.3 Word problems involving equal groups, arrays, and measurement quantities can be used to build students' understanding of and skill with multiplication and division, as well as to allow students to demonstrate their understanding of and skill with these operations.

3.OA.C.7 Finding single-digit products and related quotients is a required fluency for grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply, and divide with fractions. After multiplication and division situations have been established, reasoning about patterns in products (e.g., products involving factors of 5 or 9) can help students remember particular products and quotients. Practice — and if necessary, extra support — should continue all year for those who need it to attain fluency.

3.NF.A.2 Developing an understanding of fractions as numbers is essential for future work with the number system. It is critical that students at this grade are able to place fractions on a number line diagram and understand them as a related component of their ever-expanding number system.

3.MD.A.2 Continuous measurement quantities such as liquid volume, mass, and so on are an important context for fraction arithmetic (cf. **4.NF.B.4c, 5.NF.B.7c, 5.NF.B.3**). In grade 3, students begin to get a feel for continuous measurement quantities and solve whole-number problems involving such quantities.

3.MD.C.7 Area is a major concept within measurement, and area models must function as a support for multiplicative reasoning in grade 3 and beyond.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

- Students learn and use strategies for finding products and quotients that are based on the properties of operations; for example, to find 4×7, they may recognize that $7 = 5 + 2$ and compute $4 \times 5 + 4 \times 2$. This is an example of seeing and making use of structure (**MP.7**). Such reasoning processes amount to brief arguments that students may construct and critique (**MP.3**).

- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (**MP.1**), look for and make use of structure (**MP.7**) as they model these situations with mathematical forms (**MP.4**), and attend to precision (**MP.6**) as they distinguish different kinds of situations over time (**MP.8**).

Content Emphases by Cluster[17]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

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[17] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

■ A. Represent and solve problems involving multiplication and division.

■ B. Understand properties of multiplication and the relationship between multiplication and division.

■ C. Multiply and divide within 100.

■ D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

○ A. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions

■ A. Develop understanding of fractions as numbers.

Measurement and Data

■ A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

◘ B. Represent and interpret data.

■ C. Geometric measurement: understand concepts of area and relate area to multiplication and addition.

○ D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

◘ A. Reason with shapes and their attributes.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (**3.MD.B.3**). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.

- Reason with shapes and their attributes: Work toward meeting **3.G.A.2** should be positioned in support of area measurement and understanding of fractions.

Grade 3

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division		**Major**
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.*	
Understand properties of multiplication and the relationship between multiplication and division		**Major**
3.OA.5	Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	
3.OA.6	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	
Multiply and divide within 100		**Major**
3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	
Solve problems involving the four operations, and identify and explain patterns in arithmetic		**Major**
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]	
3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	

Grade 3

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic [4]	Additional
3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions [5] (NF)

Develop understanding of fractions as numbers	Major
3.NF.1	Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data (MD)

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects	Major
3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Grade 3

3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]	
Represent and interpret data		**Supporting**
3.MD.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition		**Major**
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	
3.MD.6	Measure areas by counting unit squares (square cm, square *m*, square *in*, square *ft*, and improvised units).	
3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures		**Additional**
3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	

Grade 3

Geometry (G)		
Reason with shapes and their attributes		**Supporting**
3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	
3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*.	

[1] See Glossary, Table 2.

[2] Students need not use formal terms for these properties.

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

[4] A range of algorithms may be used.

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

[6] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7] Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Grade 3

Additional Resources:

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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 4

In Grade 4, instruction should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Each critical area is described below.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and

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conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 3 to Grade 4

- In grade 3, students studied multiplication in terms of equal groups, arrays and area. In grade 4, students extend their concept of multiplication to make multiplicative comparisons (4.OA.A.1).[18]

- Students in grade 4 apply and extend their understanding of the meanings and properties of addition and subtraction of whole numbers to extend addition and subtraction to fractions (4.NF.B.3).[19]

- Fraction equivalence is an important theme within the standards that begins in grade 3. In grade 4, students extend their understanding of fraction equivalence to the general case, $a/b = (n \times a)/(n \times b)$ (3.NF.A.3 → 4.NF.A.1).[20] They apply this understanding to compare fractions in the general case (3.NF.A.3d → 4.NF.A.2).

- Students in grade 4 apply and extend their understanding of the meanings and properties of multiplication of whole numbers to multiply a fraction by a whole number (4.NF.B.4).

- Students in grade 4 begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass and time (4.MD.A.2).

- Students combine their understanding of the meanings and properties of multiplication and division with their understanding of base-ten units to begin to multiply and divide multidigit numbers (4.NBT.B.5–6; this builds on work done in grade 3, cf. 3.NBT.A.3).

- Students generalize their previous understanding of place value for multidigit whole numbers (4.NBT.A.1–3). This supports their work in multidigit multiplication and division, carrying forward into grade 5, when students will extend place value to decimals.

Fluency Expectations or Examples of Culminating Standards

4.NBT.B.4 Students fluently add and subtract multidigit whole numbers using the standard algorithm.

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[18] In an additive comparison problem (grades 1–2), the underlying question is *what amount would be added to one quantity to result in the other?* In a multiplicative comparison problem, the underlying question is *what factor would multiply one quantity to result in the other?*

[19] This work is limited to equal denominators in grade 4 to give students more time to build their understanding of fraction equivalence, before adding and subtracting unlike denominators in grade 5.

[20] Students who can generate equivalent fractions can also develop strategies for adding fractions with different denominators, but this is not a requirement in grade 4.

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PARCC Model Content Frameworks Indications (continued)

Examples of Major Within-Grade Dependencies

- Students' work with decimals (4.NF.C.5–7) depends to some extent on concepts of fraction equivalence and elements of fraction arithmetic. Students express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100; comparisons of decimals require that students use similar reasoning to comparisons with fractions.

- Standard 4.MD.A.2 refers to using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, time, and so on. Some parts of this standard could be met earlier in the year (such as using whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit — see also 4.MD.A.1), while others might be met only by the end of the year (such as word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number — see also 4.NF.B.3d and 4.NF.B.4c).

- Standard 4.MD.C.7 refers to word problems involving unknown angle measures. Before this standard can be met, students must understand concepts of angle measure (4.MD.C.5) and, presumably, gain some experience measuring angles (4.MD.C.6). Before that can happen, students must have some familiarity with the geometric terms that are used to define angles as geometric shapes (4.G.A.1).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do with units of measure (4.MD.A.1–2) and with multiplication of a fraction by a whole number (4.NF.B.4) can be connected to the idea of "times as much" in multiplication (4.OA.A.1).

- Addition of fractions (4.NF.B.3) and concepts of angle measure (4.MD.C.5a and 4.MD.C.7) are connected in that a one-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360.

Examples of Opportunities for In-Depth Focus

4.NBT.B.5 When students work toward meeting this standard, they combine prior understanding of multiplication with deepening understanding of the base-ten system of units to express the product of two multidigit numbers as another multidigit number. This work will continue in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NBT.B.6 When students work toward meeting this standard, they combine prior understanding of multiplication and division with deepening understanding of the base-ten system of units to find whole-number

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PARCC Model Content Frameworks Indications (continued)

quotients and remainders with up to four-digit dividends and one-digit divisors. This work will develop further in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NF.A.1 Extending fraction equivalence to the general case is necessary to extend arithmetic from whole numbers to fractions and decimals.

4.NF.B.3 This standard represents an important step in the multi-grade progression for addition and subtraction of fractions. Students extend their prior understanding of addition and subtraction to add and subtract fractions with like denominators by thinking of adding or subtracting so many unit fractions.

4.NF.B.4 This standard represents an important step in the multi-grade progression for multiplication and division of fractions. Students extend their developing understanding of multiplication to multiply a fraction by a whole number.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students decompose numbers into sums of multiples of base-ten units to multiply them (4.NBT.B.5), they are seeing and making use of structure (MP.7). As they illustrate and explain the calculation by using physical or drawn models, they are modeling (MP.4), using appropriate drawn tools strategically (MP.5) and attending to precision (MP.6) as they use base-ten units in the appropriate places.

- To compute and interpret remainders in word problems (4.OA.A.3), students must reason abstractly and quantitatively (MP.2), make sense of problems (MP.1), and look for and express regularity in repeated reasoning (MP.8) as they search for the structure (MP.7) in problems with similar interpretations of remainders.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

Content Emphases by Cluster[21]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of
a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

- ■ A. Use the four operations with whole numbers to solve problems.
- ◘ B. Gain familiarity with factors and multiples.
- ○ C. Generate and analyze patterns.

Number and Operations in Base Ten

- ■ A. Generalize place value understanding for multi-digit whole numbers.
- ■ B. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions

- ■ A. Extend understanding of fraction equivalence and ordering.
- ■ B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- ■ C. Understand decimal notation for fractions, and compare decimal fractions.

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[21] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

Measurement and Data

- A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- B. Represent and interpret data.
- C. Geometric measurement: understand concepts of angle and measure angles.

Geometry

- A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Gain familiarity with factors and multiples: Work in this cluster supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
- Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

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Operations and Algebraic Thinking (OA)

Use the four operations with whole numbers to solve problems		**Major**
4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]	
4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Gain familiarity with factors and multiples		**Supporting**
4.OA.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	
Generate and analyze patterns		**Additional**
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	

Number and Operations in Base Ten [2] (NBT)

Generalize place value understanding for multi-digit whole numbers		**Major**
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	
4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.	

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Use place value understanding and properties of operations to perform multi-digit arithmetic		**Major**
4.NBT.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	
4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	

Number and Operations—Fractions [3] (NF)

Extend understanding of fraction equivalence and ordering		**Major**
4.NF.1	Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	
4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers		**Major**
4.NF.3	Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	

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4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	
Understand decimal notation for fractions, and compare decimal fractions		**Major**
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.	
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
Measurement and Data (MD)		
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit		**Supporting**
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),…*	
4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
4.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	

Grade 4

Represent and interpret data		Supporting
4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	
Geometric measurement: understand concepts of angle and measure angles		**Additional**
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	
Geometry (G)		
Draw and identify lines and angles, and classify shapes by properties of their lines and angles		**Additional**
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	

[1] See Glossary, Table 2.

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal l to 1,000,000.

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Grade 4

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 5

In Grade 5, instruction should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. Each critical area is described below.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations,**

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Geometry, Statistics & Probability (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 4 to Grade 5

- In grade 5, students will integrate decimal fractions more fully into the place value system (5.NBT.A.1–4). By thinking about decimals as sums of multiples of base-ten units, students begin to extend algorithms for multi-digit operations to decimals (5.NBT.B.7).

- Students use their understanding of fraction equivalence and their skill in generating equivalent fractions as a strategy to add and subtract fractions, including fractions with unlike denominators.

- Students apply and extend their previous understanding of multiplication to multiply a fraction or whole number by a fraction (5.NF.B.4). They also learn the relationship between fractions and division, allowing them to divide any whole number by any nonzero whole number and express the answer in the form of a fraction or mixed number (5.NF.B.3). And they apply and extend their previous understanding of multiplication and division to divide a unit fraction by a whole number or a whole number by a unit fraction.[22]

- Students extend their grade 4 work in finding whole-number quotients and remainders to the case of two-digit divisors (5.NBT.B.6).

- Students continue their work in geometric measurement by working with volume as an attribute of solid figures and as a measurement quantity (5.MD.C.3–5).

- Students build on their previous work with number lines to use two perpendicular number lines to define a coordinate system (5.G.A.1–2).

Fluency Expectations or Examples of Culminating Standards

5.NBT.B.5 Students fluently multiply multidigit whole numbers using the standard algorithm.

Examples of Major Within-Grade Dependencies

- Understanding that in a multidigit number, a digit in one place represents $1/10$ of what it represents in the place to its left (5.NBT.A.1) is an example of multiplying a quantity by a fraction (5.NF.B.4).

[22] Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But the division of a fraction by a fraction is not a requirement in this grade.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do in multiplying fractions extends their understanding of the operation of multiplication. For example, to multiply $a/b \times q$ (where q is a whole number or a fraction), students can interpret $a/b \times q$ as meaning a parts of a partition of q into b equal parts (5.NF.B.4a). This interpretation of the product leads to a product that is less than, equal to or greater than q depending on whether $a/b < 1$, $a/b = 1$ or $a/b > 1$, respectively (5.NF.B.5).
- Conversions within the metric system represent an important practical application of the place value system. Students' work with these units (5.MD.A.1) can be connected to their work with place value (5.NBT.A.1).

Examples of Opportunities for In-Depth Focus

5.NBT.A.1 The extension of the place value system from whole numbers to decimals is a major intellectual accomplishment involving understanding and skill with base-ten units and fractions.

5.NBT.B.6 The extension from one-digit divisors to two-digit divisors requires care. This is a major milestone along the way to reaching fluency with the standard algorithm in grade 6 (6.NS.B.2).

5.NF.A.2 When students meet this standard, they bring together the threads of fraction equivalence (grades 3–5) and addition and subtraction (grades K–4) to fully extend addition and subtraction to fractions.

5.NF.B.4 When students meet this standard, they fully extend multiplication to fractions, making division of fractions in grade 6 (6.NS.A.1) a near target.

5.MD.C.5 Students work with volume as an attribute of a solid figure and as a measurement quantity. Students also relate volume to multiplication and addition. This work begins a progression leading to valuable skills in geometric measurement in middle school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students break divisors and dividends into sums of multiples of base-ten units (5.NBT.B.6), they are seeing and making use of structure (MP.7) and attending to precision (MP.6). Initially for most students, multidigit division problems take time and effort, so they also require perseverance (MP.1) and looking for and expressing regularity in repeated reasoning (MP.8).

- When students explain patterns in the number of zeros of the product when multiplying a number by powers of 10 (5.NBT.A.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).

- When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (5.MD.C.5), they also have an opportunity to look for and express regularity in repeated reasoning (MP.8). They attend to precision (MP.6) as they use correct length or volume units, and they use appropriate tools strategically (MP.5) as they understand or make drawings to show these units.

Content Emphases by Cluster[23]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[23] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◻ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

○ A. Write and interpret numerical expressions.

○ B. Analyze patterns and relationships.

Number and Operations in Base Ten

■ A. Understand the place value system.

■ B. Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations — Fractions

■ A. Use equivalent fractions as a strategy to add and subtract fractions

■ B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

◻ A. Convert like measurement units within a given measurement system.

◻ B. Represent and interpret data.

■ C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

○ A. Graph points on the coordinate plane to solve real-world and mathematical problems.

○ B. Classify two-dimensional figures into categories based on their properties.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Convert like measurement units within a given measurement system: Work in these standards supports computation with decimals. For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.

- Represent and interpret data: The standard in this cluster provides an opportunity for solving real-world problems with operations on fractions, connecting directly to both Number and Operations — Fractions clusters.

Grade 5

Operations and Algebraic Thinking (OA)

Write and interpret numerical expressions		**Additional**
5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	
5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	
Analyze patterns and relationships		**Additional**
5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	

Number and Operations in Base Ten (NBT)

Understand the place value system		**Major**
5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	
5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
5.NBT.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
5.NBT.4	Use place value understanding to round decimals to any place.	
Perform operations with multi-digit whole numbers and with decimals to hundredths		**Major**
5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	
5.NBT.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	

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5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Number and Operations—Fractions (NF)	
Use equivalent fractions as a strategy to add and subtract fractions	**Major**
5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Apply and extend previous understandings of multiplication and division to multiply and divide fractions	**Major**
5.NF.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Grade 5

5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data (MD)

Convert like measurement units within a given measurement system		**Supporting**
5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	
Represent and interpret data		**Supporting**
5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition		**Major**
5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	
5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	

Grade 5

5.MD.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry (G)

Graph points on the coordinate plane to solve real-world and mathematical problems	**Additional**
5.G.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
5.G.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Classify two-dimensional figures into categories based on their properties	**Additional**
5.G.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
5.G.4	Classify two-dimensional figures in a hierarchy based on properties.

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Grade 5

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

College- and Career- Readiness Standards for Mathematics (Grades 6-8)

Mathematics | Grade 6

In Grade 6, instruction should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. Each critical area is described below.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for

Mathematics | Grade 6 (continued)

summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 5 to Grade 6

- Students' prior understanding of and skill with multiplication, division, and fractions contribute to their study of ratios, proportional relationships and unit rates (6.RP).
- Students begin using properties of operations systematically to work with variables, variable expressions, and equations (6.EE).
- Students extend their work with the system of rational numbers to include using positive and negative numbers to describe quantities (6.NS.C.5), extending the number line and coordinate plane to represent rational numbers and ordered pairs (6.NS.C.6), and understanding ordering and absolute value of rational numbers (6.NS.C.7).
- Having worked with measurement data in previous grades, students begin to develop notions of statistical variability, summarizing and describing distributions (6.SP).

Fluency Expectations or Examples of Culminating Standards

6.NS.B.2 Students fluently divide multi-digit numbers using the standard algorithm. This is the culminating standard for several years' worth of work with division of whole numbers.

6.NS.B.3 Students fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. This is the culminating standard for several years' worth of work relating to the domains of Number and Operations in Base Ten, Operations and Algebraic Thinking, and Number and Operations — Fractions.

6.NS.A.1 Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. This completes the extension of operations to fractions.

Examples of Major Within-Grade Dependencies

- Equations of the form $px = q$ (6.EE.B.7) are unknown-factor problems; the solution will sometimes be the quotient of a fraction by a fraction (6.NS.A.1).
- Solving problems by writing and solving equations (6.EE.B.7) involves not only an appreciation of how variables are used (6.EE.B.6) and what it means to solve an equation (6.EE.B.5) but also some ability to write, read, and evaluate expressions in which letters stand for numbers (6.EE.A.2).

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

- Students must be able to place rational numbers on a number line (6.NS.C.7) before they can place ordered pairs of rational numbers on a coordinate plane (6.NS.C.8). The former standard about ordering rational numbers is much more fundamental.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with ratios and proportional relationships (6.RP) can be combined with their work in representing quantitative relationships between dependent and independent variables (6.EE.C.9).

- Plotting rational numbers in the coordinate plane (6.NS.C.8) is part of analyzing proportional relationships (6.RP.A.3a, 7.RP.A.2) and will become important for studying linear equations (8.EE.C.8) and graphs of functions (8.F).[24]

- Students use their skill in recognizing common factors (6.NS.B.4) to rewrite expressions (6.EE.A.3).

- Writing, reading, evaluating, and transforming variable expressions (6.EE.A.1–4) and solving equations and inequalities (6.EE.B.7–8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (6.G.A.2).

- Working with data sets can connect to estimation and mental computation. For example, in a situation where there are 20 different numbers that are all between 8 and 10, one might quickly estimate the sum of the numbers as $9 \times 20 = 180$.

Examples of Opportunities for In-Depth Focus

6.RP.A.3 When students work toward meeting this standard, they use a range of reasoning and representations to analyze proportional relationships.

6.NS.A.1 This is a culminating standard for extending multiplication and division to fractions.

6.NS.C.8 When students work with rational numbers in the coordinate plane to solve problems, they combine and consolidate elements from the other standards in this cluster.

6.EE.A.3 By applying properties of operations to generate equivalent expressions, students use properties of operations that they are familiar with from

[24] While not required by the standards, it might be considered valuable to expose students to time series data and to time graphs as an appealing way to work with rational numbers in the coordinate plane (6.NS.C.8). For example, students could create time graphs of temperature measured each hour over a 24-hour period in a place where, to ensure a strong connection to rational numbers, temperature values might cross from positive to negative during the night and back to positive the next day.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

previous grades' work with numbers — generalizing arithmetic in the process.

6.EE.B.7 When students write equations of the form $x + p = q$ and $px = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems.[25]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- Reading and transforming expressions involves seeing and making use of structure (MP.7). Relating expressions to situations requires making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).
- The sequence of steps in the solution of an equation is a logical argument that students can construct and critique (MP.3). Such arguments require looking for and making use of structure (MP.7) and, over time, expressing regularity in repeated reasoning (MP.8).
- Thinking about the point $(1, r)$ in a graph of a proportional relationship with unit rate r involves reasoning abstractly and quantitatively (MP.2). The graph models with mathematics (MP.4) and uses appropriate tools strategically (MP.5).
- Area, surface area, and volume present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).
- Students think with precision (MP.6) and reason quantitatively (MP.2) when they use variables to represent numbers and write expressions and equations to solve a problem (**6.EE.B.6–7**).

[25] For example, suppose Daniel went to visit his grandmother, who gave him \$5.50. Then he bought a book costing \$9.20 and had \$2.30 left. To find how much money he had before visiting his grandmother, an algebraic approach leads to the equation $x + 5.50 – 9.20 = 2.30$. An arithmetic approach without using variables at all would be to begin with 2.30, then add 9.20, then subtract 5.50. This yields the desired answer, but students will eventually encounter problems in which arithmetic approaches are unrealistically difficult and algebraic approaches must be used.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

- Working with data gives students an opportunity to use appropriate tools strategically (MP.5). For example, spreadsheets can be powerful for working with a data set with dozens or hundreds of data points.

Content Emphases by Cluster

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Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning

■ A. Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

■ A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

○ B. Compute fluently with multi-digit numbers and find common factors and multiples.

■ C. Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

Expressions and Equations

- A. Apply and extend previous understandings of arithmetic to algebraic expressions.
- B. Reason about and solve one-variable equations and inequalities.
- C. Represent and analyze quantitative relationships between dependent and independent variables.

Ratios and Proportional Reasoning

- A. Understand ratio concepts and use ratio reasoning to solve problems.

Geometry

- A. Solve real-world and mathematical problems involving area, surface area and volume.

Statistics and Probability

- A. Develop understanding of statistical variability.
- B. Summarize and describe distributions.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Solve real-world and mathematical problems involving area, surface area, and volume: In this cluster, students work on problems with areas of triangles and volumes of right rectangular prisms, which connects to work in the Expressions and Equations domain. In addition, another standard within this cluster asks students to draw polygons in the coordinate plane, which supports other work with the coordinate plane in The Number System domain.

Grade 6

Ratios and Proportional Relationships (RP)

Understand ratio concepts and use ratio reasoning to solve problems		Major
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	
6.RP.2	Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]	
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

The Number System (NS)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions		Major
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	
Compute fluently with multi-digit numbers and find common factors and multiples		**Additional**
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.	
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	

Grade 6

6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Apply and extend previous understandings of numbers to the system of rational numbers	**Major**
6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ℃ >-7℃ to express the fact that –3 ℃ is warmer than -7℃*. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* |–30| *= 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 6

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions		**Major**
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.	
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.	
6.EE.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	
6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for*.	
Reason about and solve one-variable equations and inequalities		**Major**
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	
6.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	

Grade 6

Represent and analyze quantitative relationships between dependent and independent variables		Major
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume		Supporting
6.G.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	
6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	
6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	

Statistics and Probability (SP)

Develop understanding of statistical variability		Additional
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	

Grade 6

Summarize and describe distributions		Additional
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	
6.SP.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

Grade 6

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 7

In Grade 7, instruction should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 6 to Grade 7

- In grade 6, students learned about negative numbers and the kinds of quantities they can be used to represent; they also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will add, subtract, multiply, and divide within the system of rational numbers.
- Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.A.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.A.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (e.g., if a person walks $^1/_2$ mile in each $^1/_4$ hour, the unit rate is the complex fraction ½ / ¼ miles per hour or 2 miles per hour) (7.RP.A.1); and they analyze proportional relationships in geometric figures (7.G.A.1).
- Students solve a variety of problems involving angle measure, area, surface area, and volume (7.G.B.4–6).

Fluency Expectations or Examples of Culminating Standards

7.EE.B.3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.

7.EE.B.4 In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.A.1–3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.A.1).

7.NS.A.1-2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

Examples of Major Within-Grade Dependencies

- Meeting standard 7.EE.B.3 in its entirety will involve using rational number arithmetic (7.NS.A.1–3) and percents (7.RP.A.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.

- Because rational number arithmetic (7.NS.A.1–3) underlies the problem solving detailed in 7.EE.B.3 as well as the solution of linear expressions and equations (7.EE.A.1–2, 4), this work should likely begin at or near the start of the year.

- The work leading to meeting standards 7.EE.A.1–4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (7.NS.A.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (7.NS.A.2).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).

- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

Examples of Opportunities for In-Depth Focus

7.RP.A.2 Students in grade 7 grow in their ability to recognize, represent, and analyze proportional relationships in various ways, including by using tables, graphs, and equations.

7.NS.A.3 When students work toward meeting this standard (which is closely connected to 7.NS.A.1 and 7.NS.A.2), they consolidate their skill and understanding of addition, subtraction, multiplication and division of rational numbers.

7.EE.B.3 This is a major capstone standard for arithmetic and its applications.

7.EE.B.4 Work toward meeting this standard builds on the work that led to meeting 6.EE.B.7 and prepares students for the work that will lead to meeting 8.EE.C.7.

7.G.B.6 Work toward meeting this standard draws together grades 3–6 work with geometric measurement.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students compare arithmetic and algebraic solutions to the same problem (7.EE.B.4a), they are identifying correspondences between different approaches (MP.1).
- Solving an equation such as $4 = 8(x - 1/2)$ requires students to see and make use of structure (MP.7), temporarily viewing $x - 1/2$ as a single entity.
- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.A.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).
- Proportional relationships present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.

Content Emphases by Cluster[26]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[26] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning

■ A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

■ A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations

■ A. Use properties of operations to generate equivalent expressions.

■ B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

○ A. Draw, construct and describe geometrical figures and describe the relationships between them.

○ B. Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability

◘ A. Use random sampling to draw inferences about a population.

○ B. Draw informal comparative inferences about two populations.

◘ C. Investigate chance processes and develop, use, and evaluate probability models.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.
- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.

Grade 7

Ratios and Proportional Relationships (RP)		
Analyze proportional relationships and use them to solve real-world and mathematical problems		**Major**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*	
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.	
7.RP.3	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

The Number System (NS)		
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers		**Major**
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	

Grade 7

7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.[1]
Expressions and Equations (EE)	
Use properties of operations to generate equivalent expressions	**Major**
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	**Major**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Grade 7

Geometry (G)

Draw, construct, and describe geometrical figures and describe the relationships between them		**Additional**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume		**Additional**
7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	
7.G.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	

Statistics and Probability (SP)

Use random sampling to draw inferences about a population		**Supporting**
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	
Draw informal comparative inferences about two populations		**Additional**
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	

Grade 7

7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations*. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	
Investigate chance processes and develop, use, and, evaluate probability models		**Supporting**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Grade 7

Additional Resources:

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Standards for Mathematical Practice

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2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Grade 8

In Grade 8, instruction should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square

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in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

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PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a

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PARCC Model Content Frameworks Indications (continued)

nonvertical line in the coordinate plane has equation $y = mx + b$.[27] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[28]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to

[27] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf.

[28] Note that the Geometry cluster "*Understand congruence and similarity using physical models, transparencies or geometry software*" supports high school work with congruent triangles and congruent figures.

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PARCC Model Content Frameworks Indications (continued)

grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $(y - 2)/(x - 1) = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).
- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).
- Solving an equation such as $3(x - {}^1/_2) = x + 2$ requires students to see and make use of structure (MP.7).
- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the

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PARCC Model Content Frameworks Indications (continued)

time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

The Number System	
◘	A. Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations	
■	A. Work with radicals and integer exponents.
■	B. Understand the connections between proportional relationships, lines and linear equations.
■	C. Analyze and solve linear equations and pairs of simultaneous linear equations.
Functions	
■	A. Define, evaluate and compare functions.
■	B. Use functions to model relationships between quantities.
Geometry	
■	A. Understand congruence and similarity using physical models, transparencies or geometry software.
■	B. Understand and apply the Pythagorean Theorem.
○	C. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability	
◘	A. Investigate patterns of association in bivariate data.

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PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

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The Number System (NS)		
Know that there are numbers that are not rational, and approximate them by rational numbers		**Supporting**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	
Expressions and Equations (EE)		
Work with radicals and integer exponents		**Major**
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Understand the connections between proportional relationships, lines, and linear equations		**Major**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	

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Analyze and solve linear equations and pairs of simultaneous linear equations	Major
8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.EE.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions (F)

Define, evaluate, and compare functions	Major
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities	Major
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

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8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
Geometry (G)		
Understand congruence and similarity using physical models, transparencies, or geometry software		**Major**
8.G.1	Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	
Understand and apply the Pythagorean Theorem		**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres		**Additional**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	
Statistics and Probability (SP)		
Investigate patterns of association in bivariate data		**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	

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8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

[1] Function notation is not required in Grade 8.

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Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Acceleration in Middle School

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

To prepare students for high school mathematics in eighth grade, based on guidance provided in A*ppendix A: Designing High School Mathematics Courses Based on the Common Core State Standards for Mathematics* (“Appendix A”) the MDE has developed a well-crafted sequence of **compacted courses**. The term “compacted” means to compress content, which requires a faster pace to complete, as opposed to skipping content. These compacted courses are designed for districts offering the Traditional Pathway (Algebra I – Geometry – Algebra II) high school sequence, and the other for districts using an Integrated Pathway sequence (Integrated Math I – Integrated Math II – Integrated Math III) which is commonly found internationally. A snapshot of the content standards in each Pathway is available on pages 135-136. Both Pathways are based on the idea that content should compact 3 years of content into 2 years, at most. As a result, Grades 7, 8, and 9 were compacted into Grades 7 and 8 (a 3:2 compaction). Whereas, some 8^{th} grade content is addressed in the 7th grade courses, and high school content is addressed in 8th grade.

The Compacted Traditional sequence compacts CCSSM Grade 7, CCSSM Grade 8, and high school CCSSM Algebra I into two years. Upon successfully completion of this Pathway, students will be ready for CCSSM Geometry or CCSSM Algebra II in high school. The Compacted Integrated sequencecompacts CCSSM Grade 7, CCSSM Grade 8, and CCSSM Integrated Mathematics I into two years. At the end of 8th grade, these students will be ready for CCSSM Integrated Mathematics II in high school. While the CCSS Grades K-7 effectively prepare students for algebra I in 8th grade, some standards from 8th grade have been placed in the Compacted Mathematics Grade 7 course to make the Compacted Mathematics Grade 8 courses more manageable- regardless of the Pathway chosen.

Appendix A presents a set of guidelines for the development of these compacted courses:

Acceleration in Middle School (continued)

1. Compacted courses should include the same Common Core State Standards as the non-compacted courses.

It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed.

Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning.

Research has shown discrepancies in the placement of students into “advanced” classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.

Traditionally, students taking high school mathematics in the eighth grade are expected to take a Pre-Calculus or Algebra III course in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career. (see Fourth Courses section of this paper for further detail).

SECONDARY SEQUENCE OPTIONS

Students will progress according to grade level through the sixth grade. Beginning in the seventh grade, students are given course sequence options based on academic progress, teacher recommendation, and parental consent. Below are suggested secondary course sequence options:

Suggested Secondary Course Sequence Options for Mathematics

Grade Level	OPTION 1	OPTION 2	OPTION 3
7	Grade 7	Compacted Grade 7	Compacted Grade 7
8	Grade 8	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)
9	Algebra I **or** Integrated Math I	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math II
10	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math III	Geometry **or** Integrated Math III
11	Algebra II **or** Integrated Math III	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course
12	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course

PARCC Traditional Pathway Summary Table:

ALGEBRA I—GEOMETRY—ALGEBRA II

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. Algebra I and II are adjacent so as to make the shading continuous, despite the fact that some states offer these courses a year apart.

	CCSSM Standard	A I	A II	G
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3	▪		
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1	▪		
	A-SSE.A.2	▪	▪	
	A-SSE.B.3a	▪		
	A-SSE.B.3b	▪		
	A-SSE.B.3c	▪	▪	
	A-SSE.B.4		▪	
	A-APR.A.1	▪		
	A-APR.B.2		▪	
	A-APR.B.3	▪	▪	
	A-APR.C.4		▪	
	A-APR.D.6		▪	
	A-CED.A.1	▪	▪	
	A-CED.A.2	▪		
	A-CED.A.3	▪		
	A-CED.A.4	▪		
	A-REI.A.1	▪	▪	
	A-REI.A.2		▪	
	A-REI.B.3	▪		
	A-REI.B.4a	▪		
	A-REI.B.4b	▪	▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪	▪	
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪	▪	
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪	▪	
	F-IF.B.4	▪	▪	
	F-IF.B.5	▪		
	F-IF.B.6	▪	▪	
	F-IF.C.7a	▪		
	F-IF.C.7b	▪		
	F-IF.C.7c		▪	
	F-IF.C.7e		▪	
	F-IF.C.8a	▪		
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2		▪	
	F-BF.B.3	▪	▪	
	F-BF.B.4a		▪	
	F-LE.A.1	▪		
	F-LE.A.2	▪	▪	
	F-LE.A.3	▪		
	F-LE.A.4		▪	
	F-LE.B.5	▪	▪	
	F-TF.A.1		▪	
	F-TF.A.2		▪	
	F-TF.B.5		▪	
	F-TF.C.8		▪	

	CCSSM Standard	A I	A II	G
Geometry	G-CO.A.1			▪
	G-CO.A.2			▪
	G-CO.A.3			▪
	G-CO.A.4			▪
	G-CO.A.5			▪
	G-CO.B.6			▪
	G-CO.B.7			▪
	G-CO.B.8			▪
	G-CO.C.9			▪
	G-CO.C.10			▪
	G-CO.C.11			▪
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1			▪
	G-SRT.A.2			▪
	G-SRT.A.3			▪
	G-SRT.B.4			▪
	G-SRT.B.5			▪
	G-SRT.C.6			▪
	G-SRT.C.7			▪
	G-SRT.C.8			▪
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2		▪	
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1			▪
	G-GMD.A.3			▪
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4		▪	
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	
	S-ID.B.6b	▪		
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1		▪	
	S-IC.A.2		▪	
	S-IC.B.3		▪	
	S-IC.B.4		▪	
	S-IC.B.5		▪	
	S-IC.B.6		▪	
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

PARCC Integrated Pathway Summary Table:

INTEGRATED MATH I—INTEGRATED MATH II-INTEGRATED MATH III

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course.

	CCSSM Standard	M I	M II	M III
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3		▪	
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	▪
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1a	▪		
	A-SSE.A.1b	▪	▪	
	A-SSE.A.2		▪	▪
	A-SSE.B.3a		▪	
	A-SSE.B.3b		▪	
	A-SSE.B.3c	▪		
	A-SSE.B.4			▪
	A-APR.A.1		▪	
	A-APR.B.2			▪
	A-APR.B.3			▪
	A-APR.C.4			▪
	A-APR.D.6			▪
	A-CED.A.1	▪	▪	▪
	A-CED.A.2	▪	▪	▪
	A-CED.A.3	▪		
	A-CED.A.4	▪	▪	
	A-REI.A.1		▪	▪
	A-REI.A.2			▪
	A-REI.B.3	▪		
	A-REI.B.4a		▪	
	A-REI.B.4b		▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪		
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪		▪
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪		
	F-IF.B.4	▪	▪	▪
	F-IF.B.5	▪	▪	
	F-IF.B.6	▪	▪	▪
	F-IF.C.7a	▪	▪	
	F-IF.C.7b		▪	
	F-IF.C.7c			▪
	F-IF.C.7e		▪	▪
	F-IF.C.8a		▪	
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	▪
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2	▪		
	F-BF.B.3		▪	▪
	F-BF.B.4a			▪
	F-LE.A.1a	▪		
	F-LE.A.1b	▪		
	F-LE.A.1c	▪		
	F-LE.A.2	▪		
	F-LE.A.3	▪		
	F-LE.A.4			▪
	F-LE.B.5	▪		
	F-TF.A.1			▪
	F-TF.A.2			▪
	F-TF.B.5			▪
	F-TF.C.8			▪

	CCSSM Standard	M I	M II	M III
Geometry	G-CO.A.1	▪		
	G-CO.A.2	▪		
	G-CO.A.3	▪		
	G-CO.A.4	▪		
	G-CO.A.5	▪		
	G-CO.B.6	▪		
	G-CO.B.7	▪		
	G-CO.B.8	▪		
	G-CO.C.9	▪		
	G-CO.C.10	▪		
	G-CO.C.11	▪		
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1a		▪	
	G-SRT.A.1b		▪	
	G-SRT.A.2		▪	
	G-SRT.A.3		▪	
	G-SRT.B.4		▪	
	G-SRT.B.5		▪	
	G-SRT.C.6		▪	
	G-SRT.C.7		▪	
	G-SRT.C.8		▪	
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2			▪
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1		▪	
	G-GMD.A.3		▪	
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4			▪
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	▪
	S-ID.B.6b		▪	▪
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1			▪
	S-IC.A.2			▪
	S-IC.B.3			▪
	S-IC.B.4			▪
	S-IC.B.5			▪
	S-IC.B.6			▪
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

Mathematics | High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students might learn in order to take advanced courses are included in the Advanced Mathematics Plus and Algebra III courses.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group**.**

Mathematics | High School —Number and Quantity Conceptual Category

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Algebra Conceptual Category

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Mathematics | High School—Algebra Conceptual Category (continued)

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

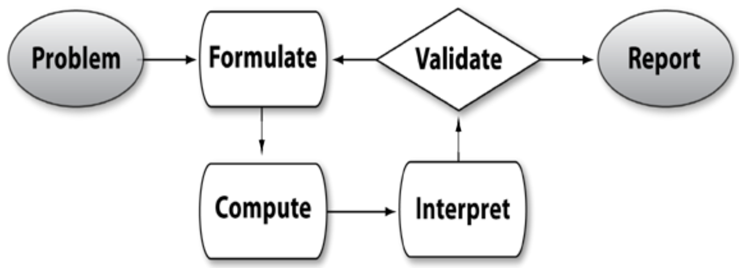
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and

Mathematics | High School—Modeling Conceptual Category (continued)

other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Mathematics | High School—Modeling Conceptual Category (continued)

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).*

Mathematics | High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many

Mathematics | High School—Geometry Conceptual Category (continued)

real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Statistics and Probability* Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematics | Compacted Mathematics Grade 7

In Compacted Mathematics Grade 7, a one-credit course, instruction should focus on four critical areas from Grade 7: (1) applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematics | Compacted Mathematics Grade 7 (continued)

In Compacted Mathematics Grade 7, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles

Mathematics | Compacted Mathematics Grade 7 (continued)

because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 6 to Grade 7

- In grade 6, students learned about negative numbers and the kinds of quantities they can be used to represent; they also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will add, subtract, multiply, and divide within the system of rational numbers.

- Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.A.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.A.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (e.g., if a person walks $^1/_2$ mile in each $^1/_4$ hour, the unit rate is the complex fraction $(^1/_2) / (^1/_4)$ miles per hour or 2 miles per hour) (7.RP.A.1); and they analyze proportional relationships in geometric figures (7.G.A.1).

- Students solve a variety of problems involving angle measure, area, surface area, and volume (7.G.B.4–6).

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Fluency Expectations or Examples of Culminating Standards

7.EE.B.3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

7.EE.B.4 In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.A.1–3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.A.1).

7.NS.A.1-2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- Meeting standard 7.EE.B.3 in its entirety will involve using rational number arithmetic (7.NS.A.1–3) and percents (7.RP.A.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

- Because rational number arithmetic (7.NS.A.1–3) underlies the problem solving detailed in 7.EE.B.3 as well as the solution of linear expressions and equations (7.EE.A.1–2, 4), this work should likely begin at or near the start of the year.

- The work leading to meeting standards 7.EE.A.1–4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (7.NS.A.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (7.NS.A.2).

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[29] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[30]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).

- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

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[29] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[30] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

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PARCC Model Content Frameworks Indications (continued)

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Examples of Opportunities for In-Depth Focus

7.RP.A.2 Students in grade 7 grow in their ability to recognize, represent, and analyze proportional relationships in various ways, including by using tables, graphs, and equations.

7.NS.A.3 When students work toward meeting this standard (which is closely connected to 7.NS.A.1 and 7.NS.A.2), they consolidate their skill and understanding of addition, subtraction, multiplication and division of rational numbers.

7.EE.B.3 This is a major capstone standard for arithmetic and its applications.

7.EE.B.4 Work toward meeting this standard builds on the work that led to meeting 6.EE.B.7 and prepares students for the work that will lead to meeting 8.EE.C.7.

7.G.B.6 Work toward meeting this standard draws together grades 3–6 work with geometric measurement.

When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students compare arithmetic and algebraic solutions to the same problem (7.EE.B.4a), they are identifying correspondences between different approaches (MP.1).
- Solving an equation such as $4 = 8(x - \frac{1}{2})$ requires students to see and make use of structure (MP.7), temporarily viewing $x - \frac{1}{2}$ as a single entity.
- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.A.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).
- Proportional relationships present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y-2)}/_{(x-1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).
- The Pythagorean Theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

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PARCC Model Content Frameworks Indications (continued)

- Solving an equation such as $3(x - \frac{1}{2}) = x + 2$ requires students to see and make use of structure (MP.7).
- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 7

Ratios and Proportional Reasoning

■ A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

■ A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations

■ A. Use properties of operations to generate equivalent expressions.

■ B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

○ A. Draw, construct and describe geometrical figures and describe the relationships between them.

○ B. Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability

◘ A. Use random sampling to draw inferences about a population.

○ B. Draw informal comparative inferences about two populations.

◘ C. Investigate chance processes and develop, use, and evaluate probability models.

Grade 8

The Number System

◘ A. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ A. Work with radicals and integer exponents.

■ B. Understand the connections between proportional relationships, lines and linear equations.

■ C. Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

Grade 8 (continued)

Functions

- A. Define, evaluate and compare functions.
- B. Use functions to model relationships between quantities.

Geometry

- A. Understand congruence and similarity using physical models, transparencies or geometry software.
- B. Understand and apply the Pythagorean Theorem.
- C. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- A. Investigate patterns of association in bivariate data.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.
- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.
- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.
- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Compacted Mathematics Grade 7

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems		**Major**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*	
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	
7.RP.3	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers		**Major**
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	

Compacted Mathematics Grade 7

7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.[1]
Know that there are numbers that are not rational, and approximate them by rational numbers	**Supporting**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Expressions and Equations	
Use properties of operations to generate equivalent expressions	**Major**
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	**Major**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

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7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	
Work with radicals and integer exponents		**Major**
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Understand the connections between proportional relationships, lines, and linear equations		**Major**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	

Compacted Mathematics Grade 7

Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
Geometry		
Draw, construct, and describe geometrical figures and describe the relationships between them		**Additional**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume		**Additional**
7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	
7.G.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	
Understand congruence and similarity using physical models, transparencies, or geometry software		**Major**
8.G.1	Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	

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8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres		**Additional**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	
Statistics and Probability		
Use random sampling to draw inferences about a population		**Supporting**
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	
Draw informal comparative inferences about two populations		**Additional**
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	
7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	

Compacted Mathematics Grade 7

Investigate chance processes and develop, use, and, evaluate probability models		**Supporting**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Compacted Mathematics Grade 7

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

In Compacted Mathematics Grade 8 (with Algebra I), a one-credit course, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles

Mathematics | Compacted Mathematics Grade 8 (with Algebra I) (continued)

because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

In Algebra I, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. Instruction should focus on five critical areas: (1) analyze and explain the process of solving equations and inequalities: (2) learn function notation and develop the concepts of domain and range; (3) use regression techniques; (4) create quadratic and exponential expressions; and (5) select from among these functions to model phenomena. Each critical area is described below.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I) (continued)

(3) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(4) In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

(5) In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4-6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (N-Q).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (6.EE.A.3, 7.EE.A.1). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (7.EE.A.2). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[31]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions.

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[31] See, for example, "Mindful Manipulation," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

- Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.

- Students in grade 8 connected their knowledge about proportional relationships, lines, and linear equations (8.EE.B.5, 6). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:

    - The graph of any linear equation in two variables is a line.

    - Any line is the graph of a linear equation in two variables.

- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.B.6). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.

- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K-8. This expands problem solving from grades K-8 dramatically.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[32] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[33]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

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[32] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[33] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y-2)}/_{(x-1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).
- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).
- Solving an equation such as $3(x - ½) = x + 2$ requires students to see and make use of structure (MP.7).

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.

- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them** (MP.1).

- **Model with mathematics** (MP.4).

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively** (MP.2). This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (A-CED, F-BF) and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.

- **Use appropriate tools strategically** (MP.5). Spreadsheets, a function modeling language, graphing tools, and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.

- **Attend to precision** (MP.6). In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (A-CED, A-REI) helps students understand the idea in new ways.

- **Look for and make use of structure** (MP.7). For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$, a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3)((7x) + 2)$ (A-SSE, A-APR).

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

- **Look for and express regularity in repeated reasoning** (MP.8). Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (A-CED). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 8

The Number System

◘ A. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ A. Work with radicals and integer exponents.

■ B. Understand the connections between proportional relationships, lines and linear equations.

■ C. Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

■ A. Define, evaluate and compare functions.

■ B. Use functions to model relationships between quantities.

Geometry

■ A. Understand congruence and similarity using physical models, transparencies or geometry software.

■ B. Understand and apply the Pythagorean Theorem.

○ C. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

◘ A. Investigate patterns of association in bivariate data.

Algebra I

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

○ B. Use properties of rational and irrational numbers (3)

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (1, <u>2</u>, 3)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (1, <u>2</u>)

◘ B. Write expressions in equivalent forms to solve problems (<u>3</u>)

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

Algebra I (continued)

Arithmetic with Polynomials and Rational Expressions (A-APR)

A. Perform arithmetic operations on polynomials (1)
B. Understand the relationship between zeros and factors of polynomials (3)

Creating Equations* (A-CED)

A. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning (1)
B. Solve equations and inequalities in one variable (3, 4)
C. Solve systems of equations (5, 6)
D. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

A. Understand the concept of a function and use function notation (1, 2, 3)
B. Interpret functions that arise in applications in terms of the context (4, 5, 6)
C. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

A. Build a function that models a relationship between two quantities (1)
B. Build new functions from existing functions (3)

Linear, Quadratic, and Exponential Models* (F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
B. Interpret expressions for functions in terms of the situation they model (5)

Interpreting Categorical and Quantitative Data (S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
B. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
C. Interpret linear models (7, 8, 9)

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (**8.NS.A.1–2**) is intimately related to work with radicals (**8.EE.A.2**), and both of these may be connected to the Pythagorean theorem (**8.G**, second cluster) as well as to volume problems (**8.G.C.9**), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Fluency Recommendations for Algebra I

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Compacted Mathematics Grade 8 (with Algebra I)

Number and Quantity

The Real Number System (N-RN)

Use properties of rational and irrational numbers		**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	

Quantities (N-Q)*

Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	

Algebra

Expressions and Expressions (EE)

Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	

Compacted Mathematics Grade 8 (with Algebra I)

Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Perform arithmetic operations on polynomials		**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Understand the relationship between zeros and factors of polynomials		**Supporting**
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	

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	Major

Solve equations and inequalities in one variable		
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Functions		
Functions (F)		
Define, evaluate, and compare functions		**Major**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]	
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	

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8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
Use functions to model relationships between quantities		**Major**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
Interpreting Functions (F-IF)		
Understand the concept of a function and use function notation		**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	

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Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	

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F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
Interpret expressions for functions in terms of the situation they model		**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
Geometry		
Geometry (G)		
Understand and apply the Pythagorean Theorem		**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
Statistics and Probability		
Statistics and Probability (SP)		
Investigate patterns of association in bivariate data		**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	

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S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

[1] Function notation is not required at Grade 8

* Modeling Standards (**High School standards only**)

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Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

In Compacted Mathematics Grade 8 (with Integrated Math I), a one-credit course, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I) (continued)

straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

In Integrated Mathematics I, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. The critical areas deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Integrated Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Each critical area is described below.

(1) By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I) (continued)

measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

(4) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(5) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(6) Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I) (continued)

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Examples of Key Advances from Grades K–8

- Students build on previous work with solving linear equations and systems of linear equations in two ways: (a) They extend to more formal solution methods, including attending to the structure of linear expressions, and (b) they solve linear inequalities.

- Students formalize their understanding of the definition of a function, particularly their understanding of linear functions, emphasizing the structure of linear expressions. Students also begin to work on exponential functions, comparing them to linear functions.

- Work with congruence and similarity motions that started in grades 6-8 progresses. Students also consider sufficient conditions for congruence of triangles.

- Work with the bivariate data and scatter plots in grades 6-8 is extended to working with lines of best fit.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

PARCC Model Content Frameworks Indications (continued)

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[34] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[35]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

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[34] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[35] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y-2)}/_{(x-1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).

- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

- Solving an equation such as $3(x - {^1/_2}) = x + 2$ requires students to see and make use of structure (MP.7).

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

PARCC Model Content Frameworks Indications (continued)\

- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.
    - **Modeling with mathematics** (MP.4) should be a particular focus as students see the purpose and meaning for working with linear and exponential equations and functions.
    - **Using appropriate tools strategically** (MP.5) is also important as students explore those models in a variety of ways, including with technology. For example, students might be given a set of data points and experiment with graphing a line that fits the data.
    - As Mathematics I continues to develop a foundation for more formal reasoning, students should engage in the practice of **constructing viable arguments and critiquing the reasoning of others** (MP.3).

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

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PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 8

The Number System	
◘	A. Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations	
■	A. Work with radicals and integer exponents.
■	B. Understand the connections between proportional relationships, lines and linear equations.
■	C. Analyze and solve linear equations and pairs of simultaneous linear equations.
Functions	
■	A. Define, evaluate and compare functions.
■	B. Use functions to model relationships between quantities.
Geometry	
■	A. Understand congruence and similarity using physical models, transparencies or geometry software.
■	B. Understand and apply the Pythagorean Theorem.
○	C. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability	
◘	A. Investigate patterns of association in bivariate data.

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PARCC Model Content Frameworks Indications (continued)

Integrated Math I
Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content.

Quantities* (N-Q)
- □ A. Reason quantitatively and use units to solve problems (1, <u>2</u>, 3)

Seeing Structure in Expressions (A-SSE)
- ■ A. Interpret the structure of expressions (<u>1</u>)
- □ B. Write expressions in equivalent forms to solve problems (<u>3</u>)

Creating Equations* (A-CED)
- ■ A. Create equations that describe numbers or relationships (<u>1</u>, <u>2</u>, 3, <u>4</u>)

Reasoning with Equations and Inequalities (A-REI)
- ■ B. Solve equations and inequalities in one variable (3)
- ○ C. Solve systems of equations (5, 6)
- ■ D. Represent and solve equations and inequalities graphically (10, <u>11</u>, 12)

Interpreting Functions (F-IF)
- ■ A. Understand the concept of a function and use function notation (1, 2, 3)
- ■ B. Interpret functions that arise in applications in terms of the context (<u>4</u>, <u>5</u>, <u>6</u>)
- □ C. Analyze functions using different representations (<u>7</u>, <u>9</u>)

Building Functions (F-BF)
- ■ A. Build a function that models a relationship between two quantities (<u>1</u>, 2)

Linear, Quadratic, and Exponential Models* (F-LE)
- □ A. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- □ B. Interpret expressions for functions in terms of the situation they model (5)

Congruence (G-CO)
- □ A. Experiment with transformations in the plane (1, 2, 3, 4, 5)
- ■ B. Understand congruence in terms of rigid motions (6, 7, 8)
- ■ C. Prove geometric theorems (9, 10, 11)

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PARCC Model Content Frameworks Indications

Integrated Math I (continued)

Interpreting Categorical and Quantitative Data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- C. Interpret linear models (7, 8, 9)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Fluency Recommendations for Integrated Math I

A/G High school students should become fluent in solving characteristic problems involving the analytic geometry of lines, such as finding the equation of a line given a point and a slope. This fluency can support students in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

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Number and Quantity

Quantities (N-Q)*

Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	

Algebra

Expressions and Expressions (EE)

Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

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Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

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Functions		
Functions (F)		
Define, evaluate, and compare functions		**Major**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]	
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
Use functions to model relationships between quantities		**Major**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
Interpreting Functions (F-IF)		
Understand the concept of a function and use function notation		**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	

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Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	

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F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
	Interpret expressions for functions in terms of the situation they model	**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
	Geometry	
	Geometry (G)	
	Understand and apply the Pythagorean Theorem	**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
	Congruence (G-CO)	
	Experiment with transformations in the plane	**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	Understand congruence in terms of rigid motions	**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	

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G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	Prove geometric theorems	**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	
	Statistics and Probability	
	Statistics and Probability (SP)	
	Investigate patterns of association in bivariate data	**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	

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Interpreting Categorical and Quantitative Data (S-ID)	
Summarize, represent, and interpret data on a single count or measurement variable	**Supporting**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*
Summarize, represent, and interpret data on two categorical and quantitative variables	**Additional**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* C. Fit a linear function for a scatter plot that suggests a linear association.
Interpret linear models	**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*
S-ID.9	Distinguish between correlation and causation.*

[1] Function notation is not required at Grade 8

* Modeling Standards (**High School standards only**)

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Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

College- and Career- Readiness Standards for Mathematics (Grades 9-12)

SECONDARY SEQUENCE OPTIONS

Students will progress according to grade level through the sixth grade. Beginning in the seventh grade, students are given course sequence options based on academic progress, teacher recommendation, and parental consent. Below are suggested secondary course sequence options:

Suggested Secondary Course Sequence Options for Mathematics

Grade Level	OPTION 1	OPTION 2	OPTION 3
7	Grade 7	Compacted Grade 7	Compacted Grade 7
8	Grade 8	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)
9	Algebra I **or** Integrated Math I	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math II
10	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math III	Geometry **or** Integrated Math III
11	Algebra II **or** Integrated Math III	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course
12	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course

PARCC TRADITIONAL PATHWAY SUMMARY TABLE:
ALGEBRA I—GEOMETRY—ALGEBRA II

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. <u>Algebra I and II are adjacent so as to make the shading continuous, despite the fact that some states offer these courses a year apart.</u>

	CCSSM Standard	A I	A II	G
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3	▪		
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1	▪		
	A-SSE.A.2	▪	▪	
	A-SSE.B.3a	▪		
	A-SSE.B.3b	▪		
	A-SSE.B.3c	▪	▪	
	A-SSE.B.4		▪	
	A-APR.A.1	▪		
	A-APR.B.2		▪	
	A-APR.B.3	▪	▪	
	A-APR.C.4		▪	
	A-APR.D.6		▪	
	A-CED.A.1	▪	▪	
	A-CED.A.2	▪		
	A-CED.A.3	▪		
	A-CED.A.4	▪		
	A-REI.A.1	▪	▪	
	A-REI.A.2		▪	
	A-REI.B.3	▪		
	A-REI.B.4a	▪		
	A-REI.B.4b	▪	▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪	▪	
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪	▪	
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪	▪	
	F-IF.B.4	▪	▪	
	F-IF.B.5	▪		
	F-IF.B.6	▪	▪	
	F-IF.C.7a	▪		
	F-IF.C.7b	▪		
	F-IF.C.7c		▪	
	F-IF.C.7e		▪	
	F-IF.C.8a	▪		
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2		▪	
	F-BF.B.3	▪	▪	
	F-BF.B.4a		▪	
	F-LE.A.1	▪		
	F-LE.A.2	▪	▪	
	F-LE.A.3	▪		
	F-LE.A.4		▪	
	F-LE.B.5	▪	▪	
	F-TF.A.1		▪	
	F-TF.A.2		▪	
	F-TF.B.5		▪	
	F-TF.C.8		▪	

	CCSSM Standard	A I	A II	G
Geometry	G-CO.A.1			▪
	G-CO.A.2			▪
	G-CO.A.3			▪
	G-CO.A.4			▪
	G-CO.A.5			▪
	G-CO.B.6			▪
	G-CO.B.7			▪
	G-CO.B.8			▪
	G-CO.C.9			▪
	G-CO.C.10			▪
	G-CO.C.11			▪
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1			▪
	G-SRT.A.2			▪
	G-SRT.A.3			▪
	G-SRT.B.4			▪
	G-SRT.B.5			▪
	G-SRT.C.6			▪
	G-SRT.C.7			▪
	G-SRT.C.8			▪
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2		▪	
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1			▪
	G-GMD.A.3			▪
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4		▪	
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	
	S-ID.B.6b	▪		
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1		▪	
	S-IC.A.2		▪	
	S-IC.B.3		▪	
	S-IC.B.4		▪	
	S-IC.B.5		▪	
	S-IC.B.6		▪	
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

PARCC INTEGRATED PATHWAY SUMMARY TABLE: INTEGRATED MATH I—INTEGRATED MATH II-INTEGRATED MATH III

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course.

	CCSSM Standard	M I	M II	M III
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3		▪	
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	▪
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1a	▪		
	A-SSE.A.1b	▪	▪	
	A-SSE.A.2		▪	▪
	A-SSE.B.3a		▪	
	A-SSE.B.3b		▪	
	A-SSE.B.3c	▪		
	A-SSE.B.4			▪
	A-APR.A.1		▪	
	A-APR.B.2			▪
	A-APR.B.3			▪
	A-APR.C.4			▪
	A-APR.D.6			▪
	A-CED.A.1	▪	▪	▪
	A-CED.A.2	▪	▪	▪
	A-CED.A.3	▪		
	A-CED.A.4	▪	▪	
	A-REI.A.1		▪	▪
	A-REI.A.2			▪
	A-REI.B.3	▪		
	A-REI.B.4a		▪	
	A-REI.B.4b		▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪		
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪		▪
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪		
	F-IF.B.4	▪	▪	▪
	F-IF.B.5	▪	▪	
	F-IF.B.6	▪	▪	▪
	F-IF.C.7a	▪	▪	
	F-IF.C.7b		▪	
	F-IF.C.7c			▪
	F-IF.C.7e		▪	▪
	F-IF.C.8a		▪	
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	▪
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2	▪		
	F-BF.B.3		▪	▪
	F-BF.B.4a			▪
	F-LE.A.1a	▪		
	F-LE.A.1b	▪		
	F-LE.A.1c	▪		
	F-LE.A.2	▪		
	F-LE.A.3	▪		
	F-LE.A.4			▪
	F-LE.B.5	▪		
	F-TF.A.1			▪
	F-TF.A.2			▪
	F-TF.B.5			▪
	F-TF.C.8			▪

	CCSSM Standard	M I	M II	M III
Geometry	G-CO.A.1	▪		
	G-CO.A.2	▪		
	G-CO.A.3	▪		
	G-CO.A.4	▪		
	G-CO.A.5	▪		
	G-CO.B.6	▪		
	G-CO.B.7	▪		
	G-CO.B.8	▪		
	G-CO.C.9	▪		
	G-CO.C.10	▪		
	G-CO.C.11	▪		
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1a		▪	
	G-SRT.A.1b		▪	
	G-SRT.A.2		▪	
	G-SRT.A.3		▪	
	G-SRT.B.4		▪	
	G-SRT.B.5		▪	
	G-SRT.C.6		▪	
	G-SRT.C.7		▪	
	G-SRT.C.8		▪	
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2			▪
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1		▪	
	G-GMD.A.3		▪	
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4			▪
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	▪
	S-ID.B.6b		▪	▪
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1			▪
	S-IC.A.2			▪
	S-IC.B.3			▪
	S-IC.B.4			▪
	S-IC.B.5			▪
	S-IC.B.6			▪
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

Mathematics | High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students might learn in order to take advanced courses are included in the Advanced Mathematics Plus and Algebra III courses. The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group**.**

Mathematics | High School—Number and Quantity Conceptual Category

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Algebra Conceptual Category

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Mathematics | High School—Algebra Conceptual Category (continued)

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

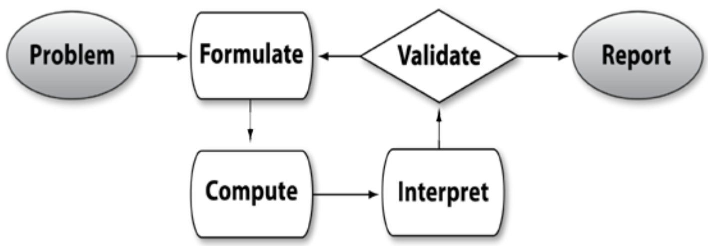
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and

Mathematics | High School—Modeling Conceptual Category (continued)

other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Mathematics | High School—Modeling Conceptual Category (continued)

Modeling Standards. *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).*

Mathematics | High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many

Mathematics | High School—Geometry Conceptual Category (continued)

real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Statistics and Probability * Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematics | High School—Algebra I Course

In Algebra I, a one-credit course, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. Instruction should focus on five critical areas: (1) analyze and explain the process of solving equations and inequalities: (2) learn function notation and develop the concepts of domain and range; (3) use regression techniques; (4) create quadratic and exponential expressions; and (5) select from among these functions to model phenomena. Each critical area is described below.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Mathematics | High School—Algebra I Course

(4) In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

(5) In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4-6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (**N-Q**).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (**6.EE.A.3**, **7.EE.A.1**). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (**7.EE.A.2**). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[36]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions. Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.

- Students in grade 8 connected their knowledge about proportional relationships, lines, and linear equations (**8.EE.B.5, 6**). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:
    - The graph of any linear equation in two variables is a line.
    - Any line is the graph of a linear equation in two variables.

[36] See, for example, "Mindful Manipulation," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., **S-ID.B.6**). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.

- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K-8. This expands problem solving from grades K-8 dramatically.

Discussion of Mathematical Practices in Relation to Course Content

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them** (MP.1).

- **Model with mathematics** (MP.4).

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively** (MP.2). This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (**A-CED, F-BF**) and then transforming the models via algebraic calculations (**A-SSE, A-APR, F-IF**) to reveal properties of the problems.

- **Use appropriate tools strategically** (MP.5). Spreadsheets, a function modeling language, graphing tools, and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.

- **Attend to precision** (MP.6). In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (**A-CED, A-REI**) helps students understand the idea in new ways.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

- **Look for and make use of structure** (MP.7). For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$, a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3)((7x) + 2)$ (**A-SSE, A-APR**).

- **Look for and express regularity in repeated reasoning** (MP.8). Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (A-CED). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans.

Fluency Recommendations

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

Content Emphases by Cluster

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

○ B. Use properties of rational and irrational numbers (3)

Quantities*(N-Q)

□ A. Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (1, 2)

□ B. Write expressions in equivalent forms to solve problems (3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ A. Perform arithmetic operations on polynomials (1)

□ B. Understand the relationship between zeros and factors of polynomials (3)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

■ A. Understand solving equations as a process of reasoning and explain the reasoning (1)

■ B. Solve equations and inequalities in one variable (3, 4)

○ C. Solve systems of equations (5, 6)

■ D. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

■ A. Understand the concept of a function and use function notation (1, 2, 3)

■ B. Interpret functions that arise in applications in terms of the context (4, 5, 6)

□ C. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

□ A. Build a function that models a relationship between two quantities (1)

○ B. Build new functions from existing functions (3)

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)

- A. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- B. Interpret expressions for functions in terms of the situation they model (5)

Interpreting categorical and quantitative data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- C. Interpret linear models (7, 8, 9)

Algebra I

Number and Quantity		
The Real Number System (N-RN)		
Use properties of rational and irrational numbers		**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

Algebra I

Arithmetic with Polynomials and Rational Expressions (A-APR)		
Perform arithmetic operations on polynomials		**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Understand the relationship between zeros and factors of polynomials		**Supporting**
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
A-REI.4	Solve quadratic equations in one variable. c. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. d. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

Algebra I

	Solve systems of equations	**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	Represent and solve equations and inequalities graphically	**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Functions

Interpreting Functions (F-IF)

	Understand the concept of a function and use function notation	**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	
	Interpret functions that arise in applications in terms of the context	**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	

Algebra I

Analyze functions using different representations		Supporting
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* b. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	

Algebra I

Interpret expressions for functions in terms of the situation they model		**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

* Modeling Standards

Algebra I

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Geometry Course

The fundamental purpose of the course in Geometry a one-credit course, is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical areas of this course include (1) building a thorough understanding of translations, reflections, and rotations; (2) developing the understanding of similarity and several theorems; (3) extension of formulas for 2-dimensional and 3-dimensional objects (4) extension of 8^{th} grade geometric concepts of lines; (5) prove basic theorems about circles; and (6) work with experimental and theoretical probability. Each critical area is described below:

(1) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(2) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

(3) Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Mathematics | High School—Geometry Course (continued)

(4) Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

(5) Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

(6) Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Previous Grades or Courses

- Because concepts such as rotation, reflection, and translation were treated in the grade 8 standards mostly in the context of hands-on activities, and with an emphasis on geometric intuition, high school Geometry will put equal weight on precise definitions.

- In grades K-8, students worked with a variety of geometric measures (length, area, volume, angle, surface area, and circumference). In high school Geometry, students apply these component skills in tandem with others in the course of modeling tasks and other substantial applications (**MP.4**).

- The skills that students develop in Algebra I around simplifying and transforming square roots will be useful when solving problems that involve distance or area and that make use the Pythagorean theorem.

- In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (**8.G.B.6–8**). In high school Geometry, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of circles (**G-GPE.A.1**).

- The algebraic techniques developed in Algebra I can be applied to study analytic geometry. Geometric objects can be analyzed by the algebraic equations that give rise to them. Some basic geometric theorems in the Cartesian plane can be proven using algebra.

Discussion of Mathematical Practices in Relation to Course Content

- **Reason abstractly and quantitatively** (**MP.2**). Abstraction is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about *all* isosceles triangles (G-CO.C.9). Quantitative reasoning in geometry involves the real numbers in an essential way: Irrational numbers show up in work with the Pythagorean theorem (G-SRT.C.8), area formulas often depend (subtly and informally) on passing to the limit and real numbers are an essential part of the definition of dilation (G-SRT.A.1). The proper use of units can help students understand the effect of dilation on area and perimeter (N-Q.A.1).

- **Construct viable arguments and critique the reasoning of others** (**MP.3**). While all of high school mathematics should work to help students see the importance and usefulness of deductive arguments, geometry is an ideal arena for developing the skill of creating and presenting proofs (G-CO.C.9.10). One reason is that conjectures about geometric phenomena are often about infinitely many cases at once — for example, *every* angle inscribed in a semicircle is a right

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators (continued)

angle — so that such results cannot be established by checking every case (G-C.A.2).

- **Use appropriate tools strategically** (**MP.5**). Dynamic geometry environments can help students look for invariants in a whole class of geometric constructions, and the constructions in such environments can sometimes lead to an idea behind a proof of a conjecture.

- **Attend to precision** (**MP.6**). Teachers might use the activity of creating definitions as a way to help students see the value of precision. While this is possible in every course, the activity has a particularly visual appeal in geometry. For example, a class can build the definition of *quadrilateral* by starting with a rough idea ("four sides"), gradually refining the idea so that it rules out figures that do not fit the intuitive idea. Another place in geometry where precision is necessary and useful is in the refinement of conjectures so that initial conjectures that are not correct can be salvaged — two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem (G-CO.B.8).

- **Look for and make use of structure** (**MP.7**). Seeing structure in geometric configurations can lead to insights and proofs. This often involves the creation of auxiliary lines not originally part of a given figure. Two classic examples are the construction of a line through a vertex of a triangle parallel to the opposite side as a way to see that the angle measures of a triangle add to 180 degrees and the introduction of a symmetry line in an isosceles triangle to see that the base angles are congruent (G-CO.C.9, 10). Another kind of hidden structure makes use of area as a device to establish results about proportions, such as the important theorem (and its converse) that a line parallel to one side of a triangle divides the other two sides proportionally (G-SRT.B.4).

Fluency Recommendations

G-SRT.B.5	Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.
G-GPE.B.4, 5, 7	Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators (continued)

G-CO.D.12 Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

Congruence (G-CO)

□ A. Experiment with transformations in the plane (1, 2, 3, 4, 5)

■ B. Understand congruence in terms of rigid motions (6, 7, 8)

■ C. Prove geometric theorems (9, 10, 11)

□ D. Make geometric constructions (12, 13)

Similarity, Right Triangles, and Trigonometry (G-SRT)

■ A. Understand similarity in terms of similarity transformations (1, 2, 3)

■ B. Prove theorems involving similarity (4, 5)

■ C. Define trigonometric ratios and solve problems involving right triangles (6, 7, 8)

Circles (G-C)

○ A. Understand and apply theorems about circles (1, 2, 3)

○ B. Find arc lengths and areas of sectors of circles (5)

Expressing Geometric Properties with Equations (G-GPE)

○ A. Translate between the geometric description and the equation for a conic section (1)

■ B. Use coordinates to prove simple geometric theorems algebraically (4, 5, 6, 7)

Geometric measurement and dimension (G-GMD)

○ A. Explain volume formulas and use them to solve problems (1, 3)

○ B. Visualize relationships between two-dimensional and three-dimensional objects (4)

Modeling with Geometry (G-MG)

■ A. Apply geometric concepts in modeling situations (1, 2, 3)

Geometry Course

Geometry		
Congruence (G-CO)		
Experiment with transformations in the plane		**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
Understand congruence in terms of rigid motions		**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Prove geometric theorems		**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	

Geometry Course

Make geometric constructions		**Supporting**
G-CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	
G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Similarity, Right Triangles, and Trigonometry (G-SRT)		
Understand similarity in terms of similarity transformations		**Major**
G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
Prove theorems involving similarity		**Major**
G-SRT.4	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Define trigonometric ratios and solve problems involving right triangles		**Major**
G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.	
G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	
Circles (G-C)		
Understand and apply theorems about circles		**Additional**
G-C.1	Prove that all circles are similar.	

Geometry Course

G-C.2	Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
Find arc lengths and areas of sectors of circles		**Additional**
G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
Use coordinates to prove simple geometric theorems algebraically		**Major**
G-GPE.4	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	
G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	
Geometric Measurement and Dimension (G-GMD)		
Explain volume formulas and use them to solve problems		**Additional**
G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	
Visualize relationships between two-dimensional and three-dimensional objects		**Additional**
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Geometry Course

Modeling with Geometry (G-MG)		
Apply geometric concepts in modeling situations		**Major**
G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	
G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	

* Modeling Standards

Geometry Course

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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In Algebra II, a one-credit course, students build on their work with linear, quadratic, and exponential functions, to extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with polynomial operations; (2) building connections between geometry and trigonometric ratios; (3) understanding of a variety of function families; and (4)explore statistical data. Each critical area is described below:

(1) Students develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(2) Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

(3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when

Mathematics | High School—Algebra II Course

knowledge of functions, statistics, and geometry is applied in a modeling context.

(4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

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PARCC Model Content Framework Indicators

Examples of Key Advances from Previous Grades or Courses

- In Algebra I, students added, subtracted, and multiplied polynomials. In Algebra II, students divide polynomials with remainder, leading to the factor and remainder theorems. This is the underpinning for much of advanced algebra, including the algebra of rational expressions.
- Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (**6.EE.B.5**). This perspective continues throughout Algebra I and Algebra II (**A-REI**).[37] "Reasoned solving" plays a role in Algebra II because the equations students encounter can have extraneous solutions (**A-REI.A.2**).
- In Algebra II, they extend the real numbers to complex numbers, and one effect is that they now have a complete theory of quadratic equations: Every quadratic equation with complex coefficients has (counting multiplicities) two roots in the complex numbers.
- In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (**8.G.B.6–8**). In Geometry, students proved theorems using coordinates (**G-GPE.B.4–7**). In Algebra II, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of conic sections (e.g., **G-GPE.A.1**).
- In Geometry, students began trigonometry through a study of right triangles. In Algebra II, they extend the three basic functions to the entire unit circle.
- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., **S-ID.B.6**). In a modeling context, they might informally fit an exponential function to a set of data, graphing the data and the model function on the same coordinate axes.

Discussion of Mathematical Practices in Relation to Course Content

While all of the mathematical practice standards are important in all three courses, four are especially important in the Algebra II course:

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[37] See, for example, "Reasoned Solving," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

PARCC Model Content Framework Indicators (continued)

- **Construct viable arguments and critique the reasoning of others** (**MP.3**). As in geometry, there are central questions in advanced algebra that cannot be answered definitively by checking evidence. There are important results about *all* functions of a certain type — the factor theorem for polynomial functions, for example — and these require general arguments (A-APR.B.2). Deciding whether two functions are equal on an infinite set cannot be settled by looking at tables or graphs; it requires arguments of a different sort (F-IF.C.8).

- **Attend to precision** (**MP.6**). As in the previous two courses, the habit of using precise language is not only a tool for effective communication but also a means for coming to understanding. For example, when investigating loan payments, if students can articulate something like, "What you owe at the end of a month is what you owed at the start of the month, plus $^1/_{12}$ of the yearly interest on that amount, minus the monthly payment," they are well along a path that will let them construct a recursively defined function for calculating loan payments (A-SSE.B.4).

- **Look for and make use of structure** (**MP.7**). The structure theme in Algebra I centered on seeing and using the structure of algebraic expressions. This continues in Algebra II, where students delve deeper into transforming expressions in ways that reveal meaning. The example given in the standards — that $x^4 - y^4$ can be seen as the difference of squares — is typical of this practice. This habit of seeing subexpressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of $x^4 - y^4$ described above can be used to show that the functions $\cos^4 x - \sin^4 x$ and $\cos^2 x - \sin^2 x$ are, in fact, equal (A-SSE.A.2).

  In addition, the standards call for attention to the structural similarities between polynomials and integers (A-APR.A.1). The study of these similarities can be deepened in Algebra II: Like integers, polynomials have a division algorithm, and division of polynomials can be used to understand the factor theorem, transform rational expressions, help solve equations, and factor polynomials.

- **Look for and express regularity in repeated reasoning** (**MP.8**). Algebra II is where students can do a more complete analysis of sequences (F-IF.A.3), especially arithmetic and geometric sequences, and their associated series. Developing recursive formulas for sequences is facilitated by the practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in algebraic symbols (F-BF.A.2). Technology can be a useful tool here: Most Computer Algebra Systems allow one to model recursive function definitions in notation that is close to standard mathematical

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PARCC Model Content Framework Indicators (continued)

notation. And spreadsheets make natural the process of taking successive differences and running totals (MP.5).

The same thinking — finding and articulating the rhythm in calculations — can help students analyze mortgage payments, and the ability to get a closed form for a geometric series lets them make a complete analysis of this topic. This practice is also a tool for using difference tables to find simple functions that agree with a set of data.

Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount (MP.1). For example, analyzing Heron's formula from geometry lets one connect the zeros of the expression to the degenerate triangles. As in Algebra I, the modeling practice is ubiquitous in Algebra II, enhanced by the inclusion of exponential and logarithmic functions as modeling tools (MP.4). Computer algebra systems provide students with a tool for modeling all kinds of phenomena, experimenting with algebraic objects (e.g., sequences of polynomials), and reducing the computational overhead needed to investigate many classical and useful areas of algebra (MP.5).

Fluency Recommendations

A-APR.D.6 This standard sets an expectation that students will divide polynomials with remainder by inspection in simple cases. For example, one can view the rational expression $\frac{x+4}{x+3}$ as $\frac{x+4}{x+3} = \frac{(x+3)+1}{x+3} = 1 + \frac{1}{x+3}$.

A-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

F-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.

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PARCC Model Content Framework Indicators (continued)

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

■ A. Extend the properties of exponents to rational exponents (1, 2)

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (2)

The Complex Number System (N-CN)

○ A. Perform arithmetic operations with complex numbers (1, 2)

○ C. Use complex numbers in polynomial identities and equations (7)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (2)

■ B. Write expressions in equivalent forms to solve problems (3, 4)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ B. Understand the relationship between zeros and factors of polynomials (2, 3)

○ C. Use polynomial identities to solve problems (4)

◘ D. Rewrite rational expressions (6)

Creating Equations* (A-CED)

◘ A. Create equations that describe numbers or relationships (1)

Reasoning with Equations and Inequalities (A-REI)

■ A. Understand solving equations as a process of reasoning and explain the reasoning (1, 2)

◘ B. Solve equations and inequalities in one variable (4)

○ C. Solve systems of equations (6, 7)

■ D. Represent and solve equations and inequalities graphically (11)

Interpreting Functions (F-IF)

◘ A. Understand the concept of a function and use function notation (3)

■ B. Interpret functions that arise in applications in terms of the context (4, 6)

◘ C. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

■ A. Build a function that models a relationship between two quantities (1, 2)

○ B. Build new functions from existing functions (3, 4a)

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems (2, 4)

B. Interpret expressions for functions in terms of the situation they model (5)

Trigonometric Functions (F-TF)

A. Extend the domain of trigonometric functions using the unit circle (1, 2)

B. Model periodic phenomena with trigonometric functions (5)

C. Prove and apply trigonometric identities (8)

Expressing Geometric Properties with Equations (G-GPE)

A. Translate between the geometric description and the equation for a conic section (2)

Interpreting categorical and quantitative data (S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable (4)

B. Summarize, represent, and interpret data on two categorical and quantitative variables (6)

Making Inferences and Justifying Conclusions (S-IC)

A. Understand and evaluate random processes underlying statistical experiments (1, 2)

B. Make inferences and justify conclusions from sample surveys, experiments and observational studies (3, 4, 5, 6)

Conditional Probability and the Rules of Probability (S-CP)

A. Understand independence and conditional probability and use them to interpret data (1, 2, 3, 4, 5)

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model (6, 7)

Algebra II

Number and Quantity		
The Real Number System (N-RN)		
Extend the properties of exponents to rational exponents		**Major**
N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{(1/3)\,3}$ to hold, so $[5^{1/3}]^3$ must equal 5.*	
N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
The Complex Number System (N-CN)		
Perform arithmetic operations with complex numbers		**Additional**
N-CN.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
N-CN.2	Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
Use complex numbers in polynomial identities and equations		**Additional**
N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2,$ thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	
Write expressions in equivalent forms to solve problems		**Major**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

Algebra II

A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Understand the relationship between zeros and factors of polynomials		**Major**
A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Use polynomial identities to solve problems		**Additional**
A-APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	
Rewrite rational expressions		**Supporting**
A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Supporting**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Solve equations and inequalities in one variable		**Supporting**
A-REI.4	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

Algebra II

	Solve systems of equations	**Additional**
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	
	Represent and solve equations and inequalities graphically	**Major**
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	

Functions

Interpreting Functions (F-IF)

	Understand the concept of a function and use function notation	**Supporting**
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	
	Interpret functions that arise in applications in terms of the context	**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
	Analyze functions using different representations	**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

Algebra II

F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth and decay.*	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
F-BF.4	Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*	
Interpret expressions for functions in terms of the situation they model		**Additional**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	

Algebra II

Trigonometric Functions (F-TF)		
Extend the domain of trigonometric functions using the unit circle		**Additional**
F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
Model periodic phenomena with trigonometric functions		**Additional**
F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	
Prove and apply trigonometric identities		**Additional**
F-TF.8	Prove the Pythagorean identity $\sin(\Theta)^2 + \cos(\Theta)^2 = 1$ and use it to find sin (Θ), cos (Θ), or tan (Θ), given sin (Θ), cos (Θ), or tan (Θ) and the quadrant of the angle.	
Geometry		
Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.2	Derive the equation of a parabola given a focus and directrix.	
Statistics and Probability*		
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	

Algebra II

Making Inferences and Justifying Conclusions (S-IC)		
Understand and evaluate random processes underlying statistical experiments		**Supporting**
S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*	
S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies		**Major**
S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*	
S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*	
S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*	
S-IC.6	Evaluate reports based on data.*	

Conditional Probability and the Rules of Probability (S-CP)		
Understand independence and conditional probability and use them to interpret data		**Additional**
S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).*	
S-CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	
S-CP.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	
S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	
S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	

Algebra II

Use the rules of probability to compute probabilities of compound events in a uniform probability model		**Additional**
S-CP.6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.*	
S-CP.7	Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.*	

* Modeling Standards

Algebra II

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

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PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics I Course

The fundamental purpose of Integrated Mathematics I, a one-credit course, is to formalize and extend the mathematics that students learned in the middle grades. The critical areas deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Integrated Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final critical area in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical focus areas of this course include (1)working with quantities to model and analyze situations; (2) exploring sequences and their relationships to functions; (3) working and translating between the various forms of linear equations and inequalities; (4) fitting data to a particular model; (5) establishing triangle congruency; and (6) verifying geometric relationships. Each critical area is described below:

(1) By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first critical focus area, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and

Mathematics | High School—Integrated Mathematics I Course (continued)

solve systems of linear equations in two variables. This critical area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

(4) This critical area builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(5) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this area, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(6) Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Grades K–8

- Students build on previous work with solving linear equations and systems of linear equations in two ways: (a) They extend to more formal solution methods, including attending to the structure of linear expressions, and (b) they solve linear inequalities.
- Students formalize their understanding of the definition of a function, particularly their understanding of linear functions, emphasizing the structure of linear expressions. Students also begin to work on exponential functions, comparing them to linear functions.
- Work with congruence and similarity motions that started in grades 6-8 progresses. Students also consider sufficient conditions for congruence of triangles.
- Work with the bivariate data and scatter plots in grades 6-8 is extended to working *with lines of best fit.*

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (**MP.4**) should be a particular focus as students see the purpose and meaning for working with linear and exponential equations and functions.
- **Using appropriate tools strategically** (**MP.5**) is also important as students explore those models in a variety of ways, including with technology. For example, students might be given a set of data points and experiment with graphing a line that fits the data.
- As Mathematics I continues to develop a foundation for more formal reasoning, students should engage in the practice of **constructing viable arguments and critiquing the reasoning of others** (**MP.3**).

Fluency Recommendations

A/G High school students should become fluent in solving characteristic problems involving the analytic geometry of lines, such as finding the equation of a line given a point and a slope. This fluency can support students in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators (continued)

G High school students should become fluent in using geometric transformation to represent the relationships among geometric objects. This fluency provides a powerful tool for visualizing relationships, as well as a foundation for exploring ideas both within geometry (e.g., symmetry) and outside of geometry (e.g., transformations of graphs).

S Students should be able to create a visual representation of a data set that is useful in understanding possible relationships among variables.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM standards in a listed domain or cluster are assessed.

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (1)

◘ B. Write expressions in equivalent forms to solve problems (3)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

■ B. Solve equations and inequalities in one variable (3)

○ C. Solve systems of equations (5, 6)

■ D. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

■ A. Understand the concept of a function and use function notation (1, 2, 3)

■ B. Interpret functions that arise in applications in terms of the context (4, 5, 6)

◘ C. Analyze functions using different representations (7, 9)

Building Functions (F-BF)

■ A. Build a function that models a relationship between two quantities (1, 2)

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)

- A. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- B. Interpret expressions for functions in terms of the situation they model (5)

Congruence (G-CO)

- A. Experiment with transformations in the plane (1, 2, 3, 4, 5)
- B. Understand congruence in terms of rigid motions (6, 7, 8)
- C. Prove geometric theorems (9, 10, 11)

Interpreting categorical and quantitative data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- C. Interpret linear models (7, 8, 9)

Integrated Mathematics I

Number and Quantity		
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	

Integrated Mathematics I

Reasoning with Equations and Inequalities (A-REI)		
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Functions

Interpreting Functions (F-IF)		
Understand the concept of a function and use function notation		**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	

Integrated Mathematics I

F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
Interpret expressions for functions in terms of the situation they model		**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	

Integrated Mathematics I

Geometry		
Congruence (G-CO)		
Experiment with transformations in the plane		**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
Understand congruence in terms of rigid motions		**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Prove geometric theorems		**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	

Integrated Mathematics I

Statistics and Probability *

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable		Supporting
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Additional**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* c. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

* Modeling Standards

Integrated Mathematics I

Additional Resources:

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Standards for Mathematical Practice

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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics II Course

The focus of Integrated Mathematics II, a one-credit course, is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Integrated Mathematics I as organized into 6 critical areas. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, bring more depth to the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical areas of this course include (1) exploring the distinction between rational and irrational numbers; (2) expending expertise of functions into piece-wise functions and quadratics; (3) focusing on the structure of expressions; (4) exploring compound events; (5) building a formal understanding of similarity; and (6) proving basic theorems about circles. Each critical area is described below:

(1) Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

(2) Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.
When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Mathematics | High School—Integrated Mathematics II Course (continued)

(3) Students begin this critical area by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

(4) Students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

(5) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

(6) In this area students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles is also explored.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Mathematics I

- Students extend their previous work with linear and exponential expressions, equations, systems of equations, and inequalities to quadratic relationships.
- A parallel extension occurs from linear and exponential functions to quadratic functions, where students also begin to analyze functions in terms of transformations.
- Building on their work with transformations, students produce increasingly formal arguments about geometric relationships, particularly around notions of similarity.

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (**MP.4**) should be a particular focus as students see the purpose and meaning for working with quadratic equations and functions, including **using appropriate tools strategically** (**MP.5**).
- As students explore a variety of ways to represent quadratic expressions, they should **look for and make use of structure** (**MP.7**).
- As their ability to create and use formal mathematical arguments grows, increased emphasis is placed on students' ability to **attend to precision** (**MP.6**), as well as to **construct viable arguments and critique the reasoning of others** (**MP.3**).

Fluency Recommendations

F/S Fluency in graphing functions (including linear, quadratic, and exponential) and interpreting key features of the graphs in terms of their function rules and a table of value, as well as recognizing a relationship (including a relationship within a data set), fits one of those classes. This forms a critical base for seeing the value and purpose of mathematics, as well as for further study in mathematics.

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

G-SRT.B.5 Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in geometric modeling.

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators (continued)

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

■ A. Extend the properties of exponents to rational exponents (1, 2)

○ B. Use properties of rational and irrational numbers (3)

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (2)

The Complex Number System (N-CN)

○ A. Perform arithmetic operations with complex numbers (1, 2)

○ C. Use complex numbers in polynomial identities and equations (7)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (1, 2)

■ B. Write expressions in equivalent forms to solve problems (3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ A. Perform arithmetic operations on polynomials (1)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (1, 2, 4)

Reasoning with Equations and Inequalities (A-REI)

■ A. Understand solving equations as a process of reasoning and explain the reasoning (1)

■ B. Solve equations and inequalities in one variable (4)

○ C. Solve systems of equations (7)

Interpreting Functions (F-IF)

■ B. Interpret functions that arise in applications in terms of the context (4, 5, 6)

◘ C. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

◘ A. Build a function that models a relationship between two quantities (1)

○ B. Build new functions from existing functions (3)

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators (continued)

Similarity, Right Triangles, and Trigonometry (G-SRT)

- A. Understand similarity in terms of similarity transformations (1, 2, 3)
- B. Prove theorems using similarity (4, 5)
- C. Define trigonometric ratios and solve problems involving right triangles (6, 7, 8)

Geometric Measurement and Dimension (G-GMD)

- A. Explain volume formulas and use them to solve problems (1, 3)

Interpreting categorical and Quantitative Data (S-ID)

- B. Summarize, represent, and interpret data on two categorical and quantitative variables (6)

Conditional Probability and Rules of Probability (S-CP)

- A. Understand independence and conditional probability and use them to interpret data (1, 2, 3, 4, 5)
- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model (6, 7)

Integrated Mathematics II

Number and Quantity

The Real Number System (N-RN)

	Extend the properties of exponents to rational exponents	**Major**
N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $[5^{1/3}]^3 = 5^{(1/3)\,3}$ *to hold, so* $[5^{1/3}]^3$ *must equal 5.*	
N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	Use properties of rational and irrational numbers	**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	

Quantities (N-Q) *

	Reason quantitatively and use units to solve problems	**Supporting**
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	

The Complex Number System (N-CN)

	Perform arithmetic operations with complex numbers	**Additional**
N-CN.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
N-CN.2	Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	Use complex numbers in polynomial identities and equations	**Additional**
N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.	

Algebra

Seeing Structure in Expressions (A-SSE)

	Interpret the structure of expressions	**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	

Integrated Mathematics II

A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.	
	Write expressions in equivalent forms to solve problems	**Major**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
	Perform arithmetic operations on polynomials	**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Creating Equations (A-CED) *		
	Create equations that describe numbers or relationships	**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
	Understand solving equations as a process of reasoning and explain the reasoning	**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
	Solve equations and inequalities in one variable	**Major**
A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

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Solve systems of equations	**Additional**
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
Functions	
Interpreting Functions (F-IF)	
Interpret functions that arise in applications in terms of the context	**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
Analyze functions using different representations	**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.*
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Integrated Mathematics II

Building Functions (F-BF)

Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	

Geometry

Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations		**Major**
G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
Prove theorems using similarity		**Major**
G-SRT.4	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Define trigonometric ratios and solve problems involving right triangles		**Major**
G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.	

Integrated Mathematics II

G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	
Geometric Measurement and Dimension (G-GMD)		
	Explain volume formulas and use them to solve problems	**Additional**
G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
	Summarize, represent, and interpret data on two categorical and quantitative variables	**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals.	
Conditional Probability and the Rules of Probability (S-CP)		
	Understand independence and conditional probability and use them to interpret data	**Additional**
S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*	
S-CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	
S-CP.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	
S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	
S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	

Integrated Mathematics II

Use the rules of probability to compute probabilities of compound events in a uniform probability model		**Additional**
S-CP.6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.*	
S-CP.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.*	

* Modeling Standards

Integrated Mathematics II

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics III Course

It is in Mathematics III, a one-credit course, that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with statistics and probability; (2) culminating work with the Fundamental Theorem of Algebra; (3) understanding of periodic phenomena; and (4) exploring function fitting. Each critical area is described below:

(1) In this area, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

(2) This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The area culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(3) Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2,or infinitely many triangles. This discussion of general triangles open up the idea

Mathematics | High School—Integrated Mathematics III Course (continued)

of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

(4) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process
of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Mathematics II

- Students begin to see polynomials as a system that has mathematical coherence, not just as a set of expressions of a specific type. An analogy to the integers can be made (including operations, factoring, etc.). Subsequently, polynomials can be extended to rational expressions, analogous to the rational numbers.
- The understandings that students have developed with linear, exponential and quadratic functions are extended to considering a much broader range of classes of functions.
- In statistics, students begin to look at the role of randomization in statistical design.

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (MP.4) continues to be a particular focus as students see a broader range of functions, including **using appropriate tools strategically** (**MP.5**).
- **Constructing viable arguments and critiquing the reasoning of others** (MP.3) continues to be a focus, as does **attention to precision** (MP.6), because students are expected to provide increasingly precise arguments.
- As students continue to explore a range of algebraic expressions, including polynomials, they should **look for and make use of structure** (MP.7).
- Finally, as students solidify the tools they need to continue their study of mathematics, a focus on **making sense of problems and persevering in solving them** (MP.1) is an essential component for their future success.

Fluency Recommendations

A/F Students should look at algebraic manipulation as a meaningful enterprise, in which they seek to understand the structure of an expression or equation and use properties to transform it into forms that provide useful information (e.g., features of a function or solutions to an equation). This perspective will help students continue to usefully apply their mathematical knowledge in a range of situations, whether their continued study leads them toward college or career readiness.

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators (continued)

M Seeing mathematics as a tool to model real-world situations should be an underlying perspective in everything students do, including writing algebraic expressions, creating functions, creating geometric models, and understanding statistical relationships. This perspective will help students appreciate the importance of mathematics as they continue their study of it.

N-Q In particular, students should recognize that much of mathematics is concerned with understanding quantities and their relationships. They should pick appropriate units for quantities being modeled, using them as a guide to understand a situation, and be attentive to the level of accuracy that is reported in a solution.

F-BF.B.3 Students should understand the effects of parameter changes and be able to apply them to create a rule modeling the function.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

Quantities* (N-Q)

□ A. Reason quantitatively and use units to solve problems (2)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (2)

■ B. Write expressions in equivalent forms to solve problems (4)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ B. Understand the relationship between zeros and factors of polynomials (2, 3)

○ C. Use polynomial identities to solve problems (4)

○ D. Rewrite rational expressions (6)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (1, 2)

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators (continued)

Reasoning with Equations and Inequalities (A-REI)

- A. Understand solving equations as a process of reasoning and explain the reasoning (1, 2)
- D. Represent and solve equations and inequalities graphically (11)

Interpreting Functions (F-IF)

- B. Interpret functions that arise in applications in terms of the context (4, 6)
- C. Analyze functions using different representations (7, 9)

Building Functions (F-BF)

- B. Build new functions from existing functions (3, 4a)

Linear, Quadratic, and Exponential Models * (F-LE)

- A. Construct and compare linear, quadratic, and exponential models and solve problems (4)

Trigonometric Functions (F-TF)

- A. Extend the domain of trigonometric functions using the unit circle (1, 2)
- B. Model periodic phenomena with trigonometric functions (5)
- C. Prove and apply trigonometric identities (8)

Congruence (G-CO)

- D. Make geometric constructions (12, 13)

Circles (G-C)

- A. Understand and apply theorems about circles (1, 2, 3)
- B. Find arc lengths and areas of sectors of circles (5)

Expressing Geometric Properties with Equations (G-GPE)

- A. Translate between the geometric description and the equation for a conic section (1, 2)
- B. Use coordinates to prove simple geometric theorems algebraically (4, 5, 6, 7)

Geometric Measurement and Dimension (G-GMD)

- B. Visualize relationships between two-dimensional and three-dimensional objects (4)

Modeling with Geometry (G-MG)

- A. Apply geometric concepts in modeling situations (1, 2, 3)

Interpreting Categorical and Quantitative Data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (4)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (6)

Making Inferences and Justifying Conclusions (S-IC)

- A. Understand and evaluate random processes underlying statistical experiments (1, 2)
- B. Make inferences and justify conclusions from sample surveys, experiments and observational studies (3, 4, 5, 6)

Integrated Mathematics III

Number and Quantity		
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	
Write expressions in equivalent forms to solve problems		**Major**
A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Understand the relationship between zeros and factors of polynomials		**Major**
A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Use polynomial identities to solve problems		**Additional**
A-APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	
Rewrite rational expressions		**Additional**
A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	

Integrated Mathematics III

Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	

Functions

Interpreting Functions (F-IF)		
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

Integrated Mathematics III

F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
F-BF.4	Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) =2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*	
Trigonometric Functions (F-TF)		
Extend the domain of trigonometric functions using the unit circle		**Additional**
F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
Model periodic phenomena with trigonometric functions		**Additional**
F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	
Prove and apply trigonometric identities		**Additional**
F-TF.8	Prove the Pythagorean identity $\sin(\Theta)^2 + \cos(\Theta)^2 = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$, given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle.	

Integrated Mathematics III

Geometry		
Congruence (G-CO)		
Make geometric constructions		**Supporting**
G-CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	
G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Circles (G-C)		
Understand and apply theorems about circles		**Additional**
G-C.1	Prove that all circles are similar.	
G-C.2	Identify and describe relationships among inscribed angles, radii, and chords*. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
Find arc lengths and areas of sectors of circles		**Additional**
G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
G-GPE.2	Derive the equation of a parabola given a focus and directrix.	
Use coordinates to prove simple geometric theorems algebraically		**Major**
G-GPE.4	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	
G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	

Integrated Mathematics III

G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	
Geometric Measurement and Dimension (G-GMD)		
Visualize relationships between two-dimensional and three-dimensional objects		**Additional**
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	
Modeling with Geometry (G-MG)		
Apply geometric concepts in modeling situations		**Major**
G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	
G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Supporting**
S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals.	
Making Inferences and Justifying Conclusions (S-IC)		
Understand and evaluate random processes underlying statistical experiments		**Supporting**
S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*	

Integrated Mathematics III

S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies		**Major**
S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*	
S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*	
S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*	
S-IC.6	Evaluate reports based on data.*	

* Modeling Standards

Integrated Mathematics III

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Advanced Mathematics Plus Course

Advanced Mathematics Plus, a one-credit course, specifies the mathematics that students should study in order to be college and career ready. The Advanced Mathematics Plus Course includes additional mathematics from the *Common Core State Standards for Mathematics* indicated by a (+). These "plus standards" will help students in advanced courses such as Calculus. This course was designed to be a fourth year Common Core math course. Throughout the duration of this course, teachers should make every effort to ensure the Standards for Mathematical Practice are addressed.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Advanced Mathematics Plus

Number and Quantity

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers	
N-CN.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane	
N-CN.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N-CN.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*
N-CN.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Use complex numbers in polynomial identities and equations	
N-CN.8	Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
N-CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N-VM)

Represent and model with vector quantities	
N-VM.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
N-VM.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N-VM.3	Solve problems involving velocity and other quantities that can be represented by vectors.

Advanced Mathematics Plus

Perform operations on vectors

N-VM.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N-VM.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\lVert c\mathbf{v} \rVert = \lvert c \rvert v$. Compute the direction of $c\mathbf{v}$ knowing that when $\lvert c \rvert v \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications

N-VM.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N-VM.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N-VM.8	Add, subtract, and multiply matrices of appropriate dimensions.
N-VM.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM.12	Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions (A-APR)

Use polynomial identities to solve problems

A-APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Advanced Mathematics Plus

Rewrite rational expressions	
A-APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Reasoning with Equations and Inequalities (A-REI)	
Solve systems of equations	
A-REI.8	Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
Functions	
Interpreting Functions (F-IF)	
Analyze functions using different representations	
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* d. Graph rational functions, indentifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
Building Functions (F-BF)	
Build a function that models a relationship between two quantities	
F-BF.1	Write a function that describes a relationship between two quantities. * c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
Build new functions from existing functions	
F-BF.4	Find inverse functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.
F-BF.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Advanced Mathematics Plus

Trigonometric Functions (F-TF)	
Extend the domain of trigonometric functions using the unit circle	
F-TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
F-TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions	
F-TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *
Prove and apply trigonometric identities	
F-TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	
Similarity, Right Triangles, and Trigonometry (G-SRT)	
Apply trigonometry to general triangles	
G-SRT.9	Derive the formula $A = ½\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G-SRT.10	Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Circles (G-C)	
Understand and apply theorems about circles	
G-C.4	Construct a tangent line from a point outside a given circle to the circle.
Expressing Geometric Properties with Equations (G-GPE)	
Translate between the geometric description and the equation for a conic section	

Advanced Mathematics Plus

G-GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Geometric Measurement and Dimension (G-GMD)	
Explain volume formulas and use them to solve problems	
G-GMD.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Statistics and Probability *	
Conditional Probability and the Rules of Probability (S-CP)	
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
S-CP.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.*
S-CP.9	Use permutations and combinations to compute probabilities of compound events and solve problems.*
Using Probability to Make Decisions (S-MD)	
Calculate expected values and use them to solve problems	
S-MD.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.*
S-MD.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.*
S-MD.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.**
S-MD.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?**

Advanced Mathematics Plus

Use probability to evaluate outcomes of decisions	
S-MD.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. * a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant*. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.**
S-MD.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
S-MD.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

* Modeling Standards

Advanced Mathematics Plus

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Algebra III Course

Algebra III, a one-credit course, includes content standards from the *2007 Mississippi Mathematics Framework Revised* Pre-Calculus course and the Common Core State Standards for Mathematics, and covers those skills and objectives necessary for success in courses higher than Algebra II and Integrated Mathematics III. Topics of study include sequences and series, functions, and higher order polynomials. Polynomial functions provide the context for higher-order investigations. Topics are addressed from a numeric, graphical, and analytical perspective. Technology is to be used to enhance presentation and understanding of concepts. The instructional approach should provide opportunities for students to work together collaboratively and cooperatively as they solve routine and non-routine problems. Communication strategies should include reading, writing, speaking, and critical listening as students present and evaluate mathematical arguments, proofs, and explanations about their reasoning. Algebra III is typically taken by students who have successfully completed Algebra II and Geometry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

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Algebra III

Number and Quantity	
Explore and illustrate the characteristics and operations connecting sequences and series	
1	Express sequences and series using recursive and explicit formulas.
2	Evaluate and apply formulas for arithmetic and geometric sequences and series.
3	Calculate limits based on convergent and divergent series.
4	Evaluate and apply infinite geometric series.
5	Extend the meaning of exponents to include rational numbers.
6	Simplify expressions with fractional exponents to include converting from radicals.
7	Factor algebraic expressions containing fractional exponents.
Algebra	
Analyze and manipulate functions	
8	Determine characteristics of graphs of parent functions (domain/range, increasing/decreasing intervals, intercepts, symmetry, end behavior, and asymptotic behavior).
9	Determine the end behavior of polynomial functions.
Use polynomial identities to solve problems	
10	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
11	Verify the Binomial Theorem by mathematical induction or by a combinatorial argument.
12	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
13	Write rational expressions in simplest form. (*For* example $\frac{x^3 - x^2 - x + 1}{x^3 + x^2 - x - 1} = \frac{x - 1}{x + 1}$) .
14	Decompose a rational function into partial fractions.
15	Determine asymptotes and holes of rational functions, explain how each was found, and relate these behaviors to continuity.
Perform operations on expressions, equations, inequalities and polynomials	
16	Add, subtract, multiply and divide rational expressions.
17	Solve polynomial and rational inequalities. Relate results to the behavior of the graphs.

Algebra III

18	Find the composite of two given functions and find the inverse of a given function. Extend this concept to discuss the identity function f(x) = x.
19	Simplify complex algebraic fractions (with/without variable expressions and integer exponents) to include expressing $\frac{f(x+h)-f(x)}{h}$ as a single simplified fraction when f(x) = $\frac{1}{1-x}$ for example.
20	Find the possible rational roots using the Rational Root Theorem.
21	Find the zeros of polynomial functions by synthetic division and the Factor Theorem.
22	Graph and solve quadratic inequalities.

Functions

	Analyze functions using different representations
23	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
24	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
	Build a function that models a relationship between two quantities
25	Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
	Build new functions from existing functions
26	Verify by composition that one function is the inverse of another.
27	Read values of an inverse function from a graph or a table, given that the function has an inverse.
28	Produce an invertible function from a non-invertible function by restricting the domain.
29	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
	Extend the domain of trigonometric functions using the unit circle
30	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
31	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Algebra III

Model periodic phenomena with trigonometric functions	
32	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
33	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Prove and apply trigonometric identities	
34	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	
Recognize, sketch, and transform graphs of functions	
35	Graph piecewise defined functions and determine continuity or discontinuities.
36	Describe the attributes of graphs and the general equations of parent functions (linear, quadratic, cubic, absolute value, rational, exponential, logarithmic, square root, cube root, and greatest integer).
37	Explain the effects of changing the parameters in transformations of functions.
38	Predict the shapes of graphs of exponential, logarithmic, rational, and piece-wise functions, and verify the prediction with and without technology.
39	Relate symmetry of the behavior of even and odd functions.
Apply trigonometry to general triangles	
40	Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
41	Prove the Laws of Sines and Cosines and use them to solve problems.
42	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Statistics and Probability	
Explore and apply fundamental principles of probability.	
43	Analyze expressions in summation and factorial notation to solve problems.
44	Prove statements using mathematical induction.

Algebra III

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Calculus Course

Calculus, a one-credit course, includes content standards from the *2007 Mississippi Mathematics Framework Revised.* This course focuses on the mathematics of change. The major focus is on differential and integral calculus. The use of graphing calculators and other technologies are major components of the course. The instructional approach should provide opportunities for students to work together collaboratively and cooperatively as they solve routine and non-routine problems. Communication strategies should include reading, writing, speaking, and critical listening as students present and evaluate mathematical arguments, proofs, and explanations about their reasoning. This one-credit course is designed for the student who has been successful in Algebra II, Integrated Mathematics III, or Algebra III.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Calculus

Number and Quantity	
Compute and determine the reasonableness of results in mathematical and real world situations	
1	Estimate limits from graphs or tables.
2	Estimate numerical derivatives from graphs or tables of data.
3	Prove statements using mathematical induction.
Algebra	
Demonstrate basic knowledge of functions, including their behavior and characteristics	
4	Predict and explain the characteristics and behavior of functions and their graphs (domain, range, increasing/decreasing intervals, intercepts, symmetry, and end behavior).
5	Investigate, describe, and determine asymptotic behavior using tables, graphs, and analytical methods
6	Determine and justify the continuity and discontinuity of functions
Evaluate limits and communicate an understanding of the limiting process	
7	Solve mathematical situations and application problems involving or using derivatives, including exponential, logarithmic, and trigonometric functions.
8	Calculate limits using algebraic methods.
9	Verify the behavior and direction of non-determinable limits.
Use the definition and formal rules of differentiation to compute derivatives	
10	State and apply the formal definition of a derivative.
11	Apply differentiation rules to sums, products, quotients, and powers of functions.
12	Use the chain rule and implicit differentiation.
13	Describe the relationship between differentiability and continuity.
Apply derivatives to find solutions in a variety of situations	
15	Define a derivative and explain the purpose/utility of the derivative.
16	Apply the derivative as a rate of change in varied contexts, including velocity, speed, and acceleration.
17	Apply the derivative to find tangent lines and normal lines to given curves at given points.
18	Predict and explain the relationships between functions and their derivatives.
19	Model rates of change to solve related rate problems.
20	Solve optimization problems.

Calculus

Employ various integration properties and techniques to evaluate integrals	
21	State and apply the First and Second Fundamental Theorem of Calculus.
22	Apply the power rule and u-substitution to evaluate indefinite integrals.
Geometry	
Use geometric concepts to gain insights into, answer questions about, and graph various implications of differentiation	
23	Demonstrate and explain the differences between average and instantaneous rates of change.
24	Apply differentiation techniques to curve sketching
25	Apply Rolle's Theorem and the Mean Value Theorem and their geometric consequences.
26	Identify and apply local linear approximations.
27	Analyze curves with attention to non-decreasing functions (monotonicity) and concavity.
Statistics and Probability	
Adapt integration methods to model situations to problems	
28	Apply integration to solve problems of area.
29	Utilize integrals to model and find solutions to real-world problems such as calculating displacement and total distance traveled.
Apply appropriate techniques, tools, and formulas to determine values for the definite integral	
30	Interpret the concept of definite integral as a limit of Riemann sums over equal subdivisions.

Calculus

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—SREB Math Ready Course

The Southern Region Education Board (SREB) Math Ready Course, a one-credit course, is designed to assist students who are in need of a fourth year mathematics preparatory course prior to entering college. This course is best suited for students who have not mastered skills needed for Advanced Placement courses. The course is built with rigor, innovative instructional strategies, and a concentration on contextual learning that departs from procedural memorization and focuses on engaging the students in a real-world context.. In short, this course targets students with weaknesses and college-ready skill gaps and re-educates them in new ways to ensure they are prepared for postsecondary-level mathematics.

The Math Ready course focuses on the key readiness standards from the *Common Core State Standards for Mathematics* as well as the eight Standards for Mathematical Practice needed for students to be ready to undertake postsecondary academic or career preparation in non-STEM fields or majors. The course addresses content standards taught throughout high school, including content from Algebra I, Geometry, and Algebra II that are essential for college and careers.

The SREB Math Ready Course consists of seven mandatory modules (or units): algebraic expressions, equations, measurement and proportional reasoning, linear functions, linear systems of equations, quadratic functions, exponential functions, and an optional module on summarizing and interpreting statistical data. While this course covers the basics in math practices and reviews the procedural steps needed to be successful in math, it is designed to be taught in a new, engaging way based heavily on conceptual teaching and learning. Eight units comprise this course. They are described below.

Unit 1: Algebraic Expressions

The algebraic expressions unit was designed to solidify student understanding of expressions while providing the students with an opportunity to have success early in the course. The recurring theme integrated in this unit focuses on engaging students using and expanding the concepts found within purposefully chosen activities. Through guided lessons, students will manipulate, create and analyze algebraic expressions, and look at the idea of whether different sets of numbers are closed under certain operations.

Unit 2: Equations

The equations unit calls for students to construct and evaluate problems that involve one or two steps while seeking understanding of how and why equations and inequalities are used in their daily lives. Students also use the structure of word problems and equations to rewrite and solve equations in different forms revealing different relationships.

Mathematics | High School—SREB Math Ready Course
(continued)

Unit 3: Measurement and Proportional Reasoning
This unit deals with unit conversions, using proportions for scaling, and area and volume. The unit requires higher-order thinking and number sense in order to get to the true intent of the standards covered. It is useful in helping students make connections with math and science or other subjects.

Unit 4: Linear Functions
This unit takes students back to the foundation of all high school mathematics—an in-depth study of linear functions. Along with allowing students to differentiate between relations that are functions and those that are not, the unit helps students specifically examine characteristics of linear functions. By looking closely at linear functions in multiple forms, students are expected to graph and write equations, as well as interpret their meaning in context of the slope and y-intercept. Students conclude with a project allowing them to collect their own data and write a line of best fit from that data.

Unit 5: Linear Systems of Equations
The systems unit deals with solving systems of linear equations. This involves helping students classify solutions (one, none or infinitely many), as well as set up and solve problems using systems of equations. Students also choose the best way to solve a system of equations and explain their solutions.

Unit 6: Quadratic Functions
This unit is an expansive look at quadratic functions: their graphs, tables and algebraic functions. It stresses multiple approaches to graphing, solving and understanding quadratics, as students explore, make conjectures and draw conclusions in group-work settings. In this unit, students explore and learn from multiple applications of quadratics. The unit assumes students have seen quadratics before but may not have a concrete, transferrable understanding of quadratic functions. The unit does not cover algebraic manipulations (multiplying and factoring), as these are addressed in previous units.

Unit 7: Exponential Functions
This unit develops students' fluency in exponential functions through varying real-life financial applications/inquiries.

Unit 8: Summarizing and Interpreting Statistical Data (*optional*)
In this unit, students further develop skills to read, analyze, and communicate (using words, tables, and graphs) relationships and patterns found in data sets of one or more variables. Students learn how to choose the appropriate statistical tools and measurements to assist in analysis, communicate results, and read and inter interpret graphs, measurements, and formulas which are crucial skills in a world overflowing with data. Students explore these concepts while modeling real contexts based on data they collect.

Mathematics | High School—SREB Math Ready Course
(continued)

School districts that are interested in offering this course should visit http://www.sreb.org/page/1684/math_ready.html to review and download course materials.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
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Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

*Note: Since AP Course Descriptions are updated regularly, please visit AP Central® (**apcentral.collegeboard.org**) to determine whether a more recent Course Description is available.*

AP courses in Calculus consist of a full high school academic year of work, and are one-credit courses comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the exam for each course. The two courses and the two corresponding exams are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still teach the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB Exam, most of the year must be devoted to the topics in differential and integral calculus. These topics are the focus of the AP Exam questions.

Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics taught in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics.

A *Calculus AB* subscore is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics. Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Exam or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies.

The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide
statements regarding their AP policies in their catalogs and on their websites.

Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

(continued)

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses that are the equivalent of four full years of high school mathematics before attempting calculus. These courses should include the study of algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies — including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study — it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals of AP Calculus AB and AP Calculus BC:

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.

Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

(continued)

- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.

- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.

- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.

- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

AP Central® (apcentral.collegeboard.org)
Interested parties can find the following Web resources at AP Central:

- AP Course Descriptions, information about the AP Course Audit, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.

- A searchable Institutes and Workshops database, providing information about professional development events.

- The Course Home Pages (apcentral.collegeboard.org/coursehomepages), which contain articles, teaching tips, activities, lab ideas, and other course-specific content contributed by colleagues in the AP community.

- Moderated electronic discussion groups (EDGs) for each AP course, provided to facilitate the exchange of ideas and practices.

Additional Resources
Teacher's Guides and Course Descriptions may be downloaded free of charge from AP Central; printed copies may be purchased through the College Board Store (store.collegeboard.org).

Advanced Placement (AP) Calculus AB
Advanced Placement (AP) Calculus BC

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics | High School—Advanced Placement (AP) Statistics Course

*Note: Since AP Course Descriptions are updated regularly, please visit AP Central® (**apcentral.collegeboard.org**) to determine whether a more recent Course Description is available.*

The AP statistics course, a one-credit course, introduces students to the major concepts and tools for collecting, analyzing and drawing conclusions from data. Students are exposed to four broad conceptual themes:

1. Exploring Data: Describing patterns and departures from patterns
2. Sampling and Experimentation: Planning and conducting a study
3. Anticipating Patterns: Exploring random phenomena using probability and simulation
4. Statistical Inference: Estimating population parameters and testing hypotheses

Students who successfully complete the course and exam may receive credit, advanced placement or both for a one-semester introductory college statistics course. This does not necessarily imply that the high school course should be one semester long. Each high school needs to determine the length of its AP Statistics course to best serve the needs of its students. The four themes are described below.

I. **Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns**.

In examining distributions of data, students should be able to detect important characteristics, such as shape, location, variability and unusual values. From careful observations of patterns in data, students can generate conjectures about relationships among variables. The notion of how one variable may be associated with another permeates almost all of statistics, from simple comparisons of proportions through linear regression. The difference between association and causation must accompany this conceptual development throughout.

II. Data must be collected according to a well-developed plan if valid information is to be obtained.

If data are to be collected to provide an answer to a question of interest, a careful plan must be developed. Both the type of analysis that is appropriate and the nature of conclusions that can be drawn from that analysis depend in a critical way on how the data was collected. Collecting data in a reasonable way, through either sampling or experimentation, is an essential step in the data analysis process.

Mathematics | High School—Advanced Placement (AP) Statistics Course (continued)

III. Probability is the tool used for anticipating what the distribution of data should look like under a given model.
Random phenomena are not haphazard: they display an order that emerges only in the long run and is described by a distribution. The mathematical description of variation is central to statistics. The probability required for statistical inference is not primarily axiomatic or combinatorial but is oriented toward using probability distributions to describe data.

IV. Statistical inference guides the selection of appropriate models.
Models and data interact in statistical work: models are used to draw conclusions from data, while the data are allowed to criticize and even falsify the model through inferential and diagnostic methods. Inference from data can be thought of as the process of selecting a reasonable model, including a statement in probability language, of how confident one can be about the selection.

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Interested parties can find the following Web resources at AP Central:

- AP Course Descriptions, information about the AP Course Audit, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.

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- Moderated electronic discussion groups (EDGs) for each AP course, provided to facilitate the exchange of ideas and practices.

Additional Resources
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Advanced Placement (AP) Statistics

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
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Compensatory Mathematics Course Description

COMPENSATORY MATHEMATICS COURSE DESCRIPTION

Students in need of instructional support, intervention or remediation may be enrolled in a Compensatory Mathematics course under the following stipulations: The Compensatory mathematics course:

1. must be taken in concert with a credit-bearing course at the same grade level;
2. includes content supportive of the accompanying credit-bearing course;
3. should make every attempt to incorporate the Standards for Mathematical Practice; and
4. may be taken as an elective, but **will not** satisfy the number of mathematics Carnegie units required for graduation.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
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PARCC Assessment Guidance

PARCC

Partnership for Assessment of Readiness for College and Careers

PARCC Calculator Policy

Approved 7/23/12

Grades 3-5 Calculator Policy

- PARCC mathematics assessments for Grades 3 – 5 will not allow for calculator usage.

Grades 6-8 Calculator Policy

- PARCC mathematics assessments for Grades 6-7 will allow for an online four function calculator with square root.
- PARCC mathematics assessments for Grade 8 will allow for an online scientific calculator.
- PARCC mathematics assessments are to be divided into calculator and non-calculator sessions, provided that the other sessions of the assessment are locked.
- The same calculator with maximum functionality is to be used for all items on calculator sessions.

High School Calculator Policy

- PARCC mathematics assessments for High School will allow for an online calculator with functionalities similar to that of a TI-84 graphing calculator.
- PARCC mathematics assessments are to be divided into calculator and non-calculator sessions, provided that the other sessions of the assessment are locked.
- The same calculator with maximum functionality is to be used for all items on calculator sessions.

PARCC
Partnership for Assessment of Readiness for College and Careers

Assessment Reference Sheet

Approved 8/9/12

Grade 5

1 mile = 5,280 feet 1 mile = 1,760 yards	1 pound = 16 ounces 1 ton = 2,000 pounds	1 cup = 8 fluid ounces 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts 1 liter = 1000 cubic centimeters

Right Rectangular Prism	$V = Bh$ or $V = lwh$

Grade 6

1 inch = 2.54 centimeters 1 meter = 39.37 inches 1 mile = 5,280 feet 1 mile = 1,760 yards 1 mile = 1.609 kilometers	1 kilometer = 0.62 mile 1 pound = 16 ounces 1 pound = 0.454 kilograms 1 kilogram = 2.2 pounds 1 ton = 2,000 pounds	1 cup = 8 fluid ounces 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts 1 gallon = 3.785 liters 1 liter = 0.264 gallons 1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Right Rectangular Prism	$V = Bh$ or $V = lwh$

PARCC Reference Sheet
(continued)

Grade 7

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Grade 8

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pythagorean Theorem	$a^2 + b^2 = c^2$

PARCC Reference Sheet
(continued)

High School

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	$1\ radian = \frac{180}{\pi}\ degree$
Degrees	$1\ degree = \frac{\pi}{180} radian$
Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$

Additional Resources

PARCC Model Content Frameworks

http://www.parcconline.org/parcc-model-content-frameworks

PARCC developed the Model Content Frameworks to help (1) inform development of item specifications and blueprints for the PARCC assessments, and (2) support implementation of the Common Core State Standards.

PARCC Calculator and Reference Sheet Policies

http://www.parcconline.org/assessment-administration-guidance

The PARCC reference sheets for grades 3 – High School have been developed based on the intent of the Common Core State Standards for Mathematics. One will notice that the names of the measurement formulas provided on the reference sheet only include the name of the figure or object in which the measurement formula pertains to. The intent of the CCSSM at grades 5 – 8 is to know and apply the measurement formulas. In order for students to be able to choose the correct formula, they will need to know the formula.

PARCC Sample Assessment Items

http://www.parcconline.org/samples/item-task-prototypes

The primary purpose of the PARCC sample assessment items is to provide information about the assessment system and support educators as they transition to the CCSSM and the PARCC assessments. The samples presented are designed to shed light on important elements of the CCSSM and to show how critical content in the standards may appear in PARCC's next-generation, technology-based assessments.

PARCC Performance Level Descriptors (PLDs)

http://www.parcconline.org/math-plds

The PARCC PLDs articulate the knowledge, skills, and practices that students performing at a given level should be able to demonstrate in each content area at each grade level. The ***grade- and subject-specific PLDs*** are intended to serve several purposes, including the following: (1) communicate expectations to educators about what types of performances will be necessary at the high school-level for students to demonstrate that they are college- and career-ready (CCR) or making adequate progress to become CCR; (2) communicate expectations to educators about what types of performance will be necessary in grades 3-8 for students to demonstrate that they are

academically prepared to engage successfully in further studies in each content area; (3) provide information to local educators for use in developing curricular and instructional materials; (4) serve as the basis for PARCC standard setting in summer 2015; and (5) inform item and rubric development for the PARCC assessments.

Additional Resources (continued)

PARCC Assessment Blueprints/Evidence Tables
http://www.parcconline.org/assessment-blueprints-test-specs
Blueprints are a series of documents that together describe the content and structure of an assessment. These documents define the total number of tasks and/or items for any given assessment component, the standards measured, the item types, and the point values for each. *Evidence statement tables and evidence statements* describe the knowledge and skills that an assessment item or a task elicits from students. These are aligned directly to the Common Core State Standards, and highlight their advances especially around the coherent nature of the standards.

Glossary

Glossary

Note: The words that are defined here pertain to courses derived from the *Common Core State Standards for Mathematics*.

- **Addition and subtraction within 5, 10, 20, 100, or 1000**. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

- **Additive inverses**. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

- **Associative property of addition**. See Table 3 in this Glossary.

- **Associative property of multiplication.** See Table 3 in this Glossary.

- **Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

- **Box plot**. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

- **Commutative property**. See Table 3 in this Glossary.

- **Complex fraction**. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

- **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

- **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

- **Congruent**. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

- **Counting on**. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Glossary (continued)

- **Dot plot.** *See:* line plot.

- **Dilation**. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

- **Expanded form**. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

- **Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

- **First quartile**. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

- **Fraction**. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

- **Identity property of 0**. See Table 3 in this Glossary.

- **Independently combined probability models**. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

- **Integer**. A number expressible in the form *a* or –*a* for some whole number *a*.

- **Interquartile Range**. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

- **Line plot**. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

- **Mean**. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Glossary (continued)

- **Mean absolute deviation**. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

- **Median**. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

- **Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

- **Multiplication and division within 100**. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

- **Multiplicative inverses**. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

- **Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

- **Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

- **Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

- **Properties of operations**. See Table 3 in this Glossary.

- **Properties of equality**. See Table 4 in this Glossary.

- **Properties of inequality**. See Table 5 in this Glossary.

- **Properties of operations**. See Table 3 in this Glossary.

Glossary (continued)

- **Probability**. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

- **Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

- **Random variable.** An assignment of a numerical value to each outcome in a sample space.

- **Rational expression.** A quotient of two polynomials with a non-zero denominator.

- **Rational number**. A number expressible in the form *a*/*b* or – *a*/*b* for some fraction *a*/*b*. The rational numbers include the integers.

- **Rectilinear figure.** A polygon all angles of which are right angles.

- **Rigid motion.** A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

- **Repeating decimal**. The decimal form of a rational number. *See also:* terminating decimal.

- **Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered.

- **Scatter plot**. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

- **Similarity transformation**. A rigid motion followed by a dilation.

- **Tape diagram**. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Glossary (continued)

- **Terminating decimal.** A decimal is called terminating if its repeating digit is 0.
- **Third quartile**. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.
- **Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.
- **Uniform probability model**. A probability model which assigns equal probability to all outcomes. *See also:* probability model*.*
- **Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.
- **Visual fraction model.** A tape diagram, number line diagram, or area model.
- **Whole numbers**. The numbers 0, 1, 2, 3, ….

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

[4] To be more precise, this defines the *arithmetic mean*.

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Tables 1-5

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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[6] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp.32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Source Code: *Miss. Code Ann.* § 37-1-3 (Adopted 02/2014)

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2014
Mississippi
College- and Career-
Readiness Standards
for Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Effective Date: 2014-2015 School Year

Mississippi College- and Career- Readiness Standards for Mathematics

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# Course includes content from the *2007 Mississippi Mathematics Framework Revised* and the *Common Core State Standards for Mathematics*

INTRODUCTION

Mission Statement

The Mississippi Department of Education is dedicated to student success including the improvement of student achievement in mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. The *2014 Mississippi College- and Career- Readiness Standards for Mathematics* ("The Standards") provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level and course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy.

Purpose

In an effort to closely align instruction for students who are progressing toward postsecondary study and the workforce, the *2014 Mississippi College- and Career- Readiness Standards for Mathematics* include grade- and course-specific standards for K-12 mathematics.

The primary purpose of this document is to provide a basis for curriculum development for K-12 mathematics teachers, outlining what students should know and be able to do by the end of each grade level and course. Courses for grades K-12 are based on the *Common Core State Standards for Mathematics* and *Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards for Mathematics* (www.corestandards.org). These courses also include the *PARCC Model Content Frameworks* (www.parcconline.org/parcc-model-content-frameworks) to support implementation of the standards and assessments. Mississippi-specific courses that were revised to align with the *Common Core State Standards for Mathematics* include Pre-Calculus (renamed Algebra III) and Calculus.

The Southern Regional Education Board (SREB) Math Ready course is included as a transition to college mathematics course.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Implementation

The required year for the *2014 Mississippi College- and Career- Readiness Standards for Mathematics* is school year 2014-2015.

Technology

The Mississippi Department of Education (MDE) strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The appropriate use of instructional technology is integrated throughout the *2014 Mississippi College- and Career- Readiness Standards for Mathematics.* Teaching strategies at each grade level and in every secondary course incorporate technology in the form of calculators, software, or on-line internet resources. The graphing calculator is an integral part of mathematics courses beginning with Grade 6.

The MDE believes strongly in the Principles and Standards for School Mathematics Technology Principle of the National Council for Teachers of Mathematics (NCTM):

> "Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. In the mathematics classrooms envisioned in *Principles and Standards*, every student has access to technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking."
> (NCTM, 2013, http://www.nctm.org.)

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Common Core State Standards for Mathematics Overview

COMMON CORE STATE STANDARDS FOR MATHEMATICS OVERVIEW

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge. Aiming for clarity and specificity, these Standards endeavor to follow a design that not only stresses conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific expectations but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary for college and/or careers. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with reading disabilities the use of Braille, screen reader technology, or other assistive devices should be made available. In addition, while writing, these students should have access to a scribe, computer, or speech-to-text technology in their classroom. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of College- and Career- Readiness for all students.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with

data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the 2014 Mississippi College- and Career- Readiness Standards for Mathematics

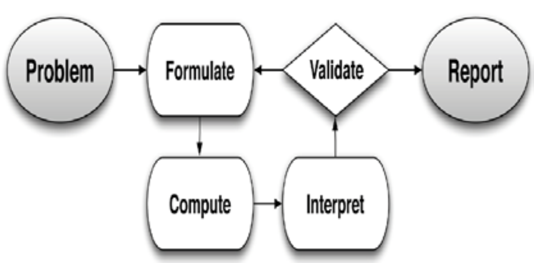
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to the Standards in mathematics instruction.

The Standards are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Modeling (High School Courses only)

Modeling standards are noted throughout the high school courses with an asterisk (*). *Modeling* links classroom mathematics and statistics to everyday life, work, and decision-making. *Modeling* is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.



Making mathematical models is a Standard for Mathematical Practice, and specific *Modeling* standards appear throughout the high school standards. The basic modeling cycle above involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

Partnership for Assessment of Readiness of College and Careers (PARCC)

Overview of PARCC

The Partnership for Assessment of Readiness for College and Careers (PARCC) is a consortium of 19 states plus the District of Columbia and the U.S. Virgin Islands working together to develop a common set of K-12 assessments in English and math anchored in what it takes to be ready for college and careers. These new K-12 assessments will build a pathway to College- and Career- Readiness by the end of high school, mark students' progress toward this goal from 3rd grade up, and provide teachers with timely information to inform instruction and provide student support. Mississippi is a governing member of PARCC and maintains a variety of positions within the PARCC infrastructure.

The PARCC Vision

PARCC states have committed to building a K-12 assessment system that:

- Builds a pathway to College- and Career- Readiness for all students,
- Creates high-quality assessments that measure the full range of the Common Core State Standards,
- Supports educators in the classroom,
- Makes better use of technology in assessments, and
- Advances accountability at all levels.

The PARCC Assessments

PARCC represents a fundamental shift in how we think about testing, state to state. PARCC is based on the core belief that assessment should not be a process to penalize educators and districts, but rather a tool for enhancing teaching and learning. The PARCC assessments in mathematics are carefully crafted to give teachers, schools, students and parents better and more useful information on how they are preparing students for careers and college — and life. Educators, specialists, and administrators within each PARCC state voluntarily work(ed) together to develop a common set of K-12 assessments in mathematics.

PARCC is committed to developing tests worth taking and are the type of new generation assessment that teachers have requested for years. In math, students will have to solve complex problems, show their work, and demonstrate how they solved the problem. Unlike pencil-and-paper bubble tests, these new assessments will more closely resemble high-quality classroom work. PARCC will measure what children are learning, in a more meaningful way.

PARCC's estimated testing time is reasonable and reflects the shift from traditional multiple choice tests to performance-based assessments. The new assessments are designed to measure the full range of knowledge and skills students need to be career and college ready or on track toward that goal, through performance based tasks. The assessments will be innovative in design and more engaging for students.

PARCC assessments will measure the full range of student performance, including the performance of high- and low-achieving students. As a governing member of PARCC, Mississippi plays a key role in all implementation and policy decisions regarding the PARCC assessments.

Purpose of the PARCC Model Content Frameworks

Included in the Introduction page of each grade/course are key features of the PARCC Model Content Frameworks, Grades K-12. As part of its proposal to the U.S. Department of Education, PARCC committed to developing model content frameworks for mathematics to serve as a bridge between the Common Core State Standards and the PARCC assessments. PARCC developed the PARCC Model Content Frameworks to: (1) inform development of item specifications and blueprints for the PARCC assessments, and (2) support implementation of the *Common Core State Standards for Mathematics.*

The PARCC Model Content Frameworks were developed through a state-led process that included mathematics content experts in PARCC member states and members of the Common Core State Standards writing team. Although the primary purpose of the Model Content Frameworks is to provide a frame for the PARCC assessments, they also are voluntary resources to help educators and those developing curricula and instructional materials.

Guidance from the PARCC Model Content Frameworks is written with the expectation that students develop content knowledge, conceptual understanding and expertise with the Standards for Mathematical Practice. The Introduction of each grade level or course standards in the *2014 Mississippi College- and Career- Readiness Standards for Mathematics* includes the following *Examples* from the PARCC Model Content Frameworks include:

- *Examples of key advances from the previous grade;*
- *Fluency expectations or examples of culminating standards;*
- *Examples of major within-grade dependencies;*
- *Examples of opportunities for connections among standards, clusters or domains;*
- *Examples of opportunities for in-depth focus;*
- *Examples of opportunities for connecting mathematical content and mathematical practices;*
- *Content emphases by cluster; and*
- *Fluency recommendations).*

Connections to the PARCC Assessment

The PARCC Assessment System will be designed to measure the knowledge, skills, and understandings essential to achieving college and career readiness. In mathematics, this includes conceptual understanding, procedural skill and fluency, and application and problem solving, as defined by the standards. Each of these works in conjunction with the others to promote students' achievement in mathematics. To measure the full range of the standards, the assessments will include tasks that require students to connect mathematical content and mathematical practices.

The Model Content Frameworks for Mathematics reflect these priorities by providing detailed information about selected practice standards, fluencies, connections, and content emphases. These emphases will be reflected in the PARCC Assessment System.

The Model Content Frameworks do not contain a suggested scope and sequence by quarter. Rather, they provide examples of key content dependencies (where one concept ought to come before another), key instructional emphases, opportunities for in-depth work on key concepts, and connections to critical practices. These last two components, in particular, intend to support local and state efforts to deliver instruction that connects content and practices while achieving the standards' balance of conceptual understanding, procedural skill and fluency, and application.

Overall, the PARCC Assessment System will include a mix of items, including short- and extended-response items, performance-based tasks, and technology-enhanced items. In mathematics, the items will be designed to elicit evidence of whether students can:

- Solve problems involving the Major work of the grade with connections to the practice standards;
- Solve problems involving the Additional and Supporting work of the grade with connections to the practice standards;
- Express mathematical reasoning by constructional mathematical arguments and critiques;
- Solve real-world problems by engaging particularly in the modeling practice; and
- Demonstrate fluency in the areas set for in the content standards for Grades 3–6.

Questions asked will measure student learning within and across various mathematical domains and practices. The questions will cover the full range of mathematics, including conceptual understanding, procedural fluency, and the varieties of expertise described by the practice standards. Mathematical understanding, procedural skill, and the ability to apply what one knows are equally important and can be assessed using mathematical tasks of sufficient richness, which PARCC will include in its assessment system.

It is critical that all students are able to demonstrate mastery of the skills and knowledge described in the standards. PARCC recognizes the importance of equity, access and fairness in its assessments and aligned materials.

PARCC Assessment Emphases

Each tested grade level or course describes cluster emphases for content standards. These are provided because curriculum, instruction, and assessment in each course must reflect the focus and emphasis of the grade level/course standards. To make relative emphases in the Standards more transparent and useful, cluster headings are designated as **Major**, **Additional** and **Supporting**. Key: ■ Major Clusters; □ or ■ Supporting Clusters; and ○ or ■ Additional Clusters.

Some clusters that are not major emphases in themselves are designed to *support* and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade would fairly be called *additional*. To say that some things have greater emphasis is not to say that anything in the Standards can be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

All standards figure in a mathematical education and therefore will be *eligible* for inclusion on the PARCC end-of-year assessments. The assessments will mirror the message that is communicated here: Major clusters will be a majority of the assessment, Supporting clusters will be assessed through their success at supporting the Major clusters and Additional clusters will be assessed as well. The assessments will strongly focus where the Standards strongly focus.

Finally, the following are some recommendations for using the cluster-level emphases:

Do …

- Use the guidance to inform instructional decisions regarding time and other resources spent on clusters of varying degrees of emphasis.

- Allow the focus on the major work of the grade to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.

- Evaluate instructional materials taking the cluster-level emphases into account. The major work of the grade must be presented with the highest possible quality; the supporting work of the grade should indeed support the major focus, not detract from it.

- Set priorities for other implementation efforts taking the emphases into account, such as staff development; new curriculum development; or revision of existing formative or summative testing at the district or school level.

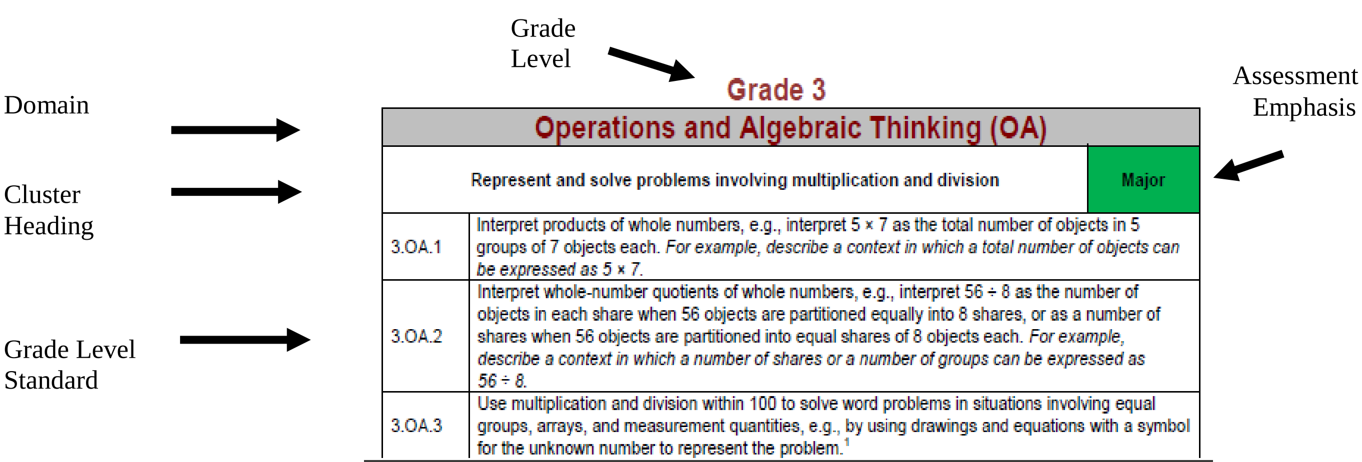
Don't …

- Neglect any material in the standards. (Instead, use the information provided to connect Supporting Clusters to the other work of the grade.)

- Sort clusters from Major to Supporting, and then teach them in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.
- Use the cluster headings as a replacement for the standards. All features of the standards matter — from the practices to surrounding text to the particular wording of individual content standards. Guidance is given at the cluster level as a way to talk about the content with the necessary specificity yet without going so far into detail as to compromise the coherence of the standards.

Reading the College- and Career- Readiness Standards

- Standards define what students should understand and be able to do.
- Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
- Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.
- Conceptual Categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.
- Assessment Emphasis is included. Major clusters will be a majority of the assessment, supporting clusters will be assessed through their success at supporting the major clusters and additional clusters will be assessed as well.



Grade 3

Operations and Algebraic Thinking (OA)		
Represent and solve problems involving multiplication and division		Major
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	

Course Title

Conceptual Category
(HS courses only)

Domain

Cluster Heading

Course Standard

Assessment Emphasis

Integrated Mathematics I

Geometry	
Congruence (G-CO)	
Experiment with transformations in the plane	Supporting
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

College- and Career- Readiness Standards for Mathematics (Grades K-5)

Mathematics | Grade K

In Kindergarten, instruction should focus on two critical areas: (1) representing, relating, and operating on whole numbers- initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics. Each critical area is described below.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics |Grade K

PARCC Model Content Frameworks Indications

Examples of Key Advances in Kindergarten

Students with academically oriented families and/or preschool experience may enter Kindergarten able to recite number words and use counting to answer "how many?" questions. But not all children have these background experiences. Fewer still will know what addition and subtraction do, let alone finding sums and differences. Kindergarten establishes these and other foundations and begins the process of building the mathematical habits of mind that lead to proficient mathematical practice and are described in the Standards for Mathematical Practice.

- Students learn to pair objects 1-1 with counting words, and they learn that the last number word tells the number of objects in a collection (up to 20). This is called "cardinal counting," as opposed to "rote counting" (merely being able to recite the counting words in order).
- Some students will progress from the "counting all" strategy to the more sophisticated strategy of "counting on" during Kindergarten (see K.CC.B.4c and 1.OA.C.5).
- Students learn to compare the number of objects in one group versus another group, and eventually to compare written numerals 1–10.
- Students understand addition as joining collections and adding to collections, and they understand subtraction as taking collections apart or taking from collections, representing these operations in a variety of ways.

Fluency Expectations or Examples of Culminating Standards

K.CC.A.3 This standard refers to written numerals from 0–20. In particular, students should work throughout the year toward fluency in writing the numerals 0–10.[38]

K.CC.B.5 This standard refers to cardinal counting and producing a collection with a given number of objects. Students should become fluent in cardinal counting well before the end of Kindergarten, because much else in Kindergarten depends on it.

[38] "Learning to write number symbols (numerals) is a much more difficult task than is reading them and often is not begun until kindergarten. Writing numerals requires children to have an accurate mental image of the symbol, which entails left-right orientation, and a motor plan to translate the mental image into the correct sequence of motor actions to form a numeral…. Some numerals are much easier than others. The loops in 6 and 9, the curve and straight line in the 2, and the crossovers in the 8 are difficult but can be mastered by kindergarten children with effort. The easier numerals 1, 3, 4, 5, and 7 can often be mastered earlier. Whenever children do learn to write numerals, learning to write correct and readable numerals is not enough. They must become fluent at writing numerals (i.e., writing numerals must become over-learned) so that writing them as part of a more complex task is not so slow or effortful as to be discouraging when solving several problems. It is common for children at this step and even later to reverse some numerals (such as 3) because the left-right orientation is difficult for them. This will become easier with age and experience." *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council (2009), p. 138.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

K.CC.C.7 Compare two numbers 1–10 presented as written numerals.

K.OA.A.5 Fluently add and subtract within 5. That is, given any two numbers 0–5 with sum less than or equal to 5, students can name the sum reasonably quickly; and likewise for related differences, given one number and a goal that is 5 or less, they can reasonably quickly name the "missing amount."

Examples of Major Within-Grade Dependencies

Standards listed later in a given domain may often refer to skills that depend on standards listed earlier in the domain. However, there are many connections between skills, within and across domains. (See some Opportunities for Connections below.) Regardless of the logical ordering of the standards, *teaching* need not (and should not) tick off standards one by one in order. For example, the "later standards" may suggest interesting situations that call for counting skills that the children are still developing, and that motivate that development. And many fluencies and skills must typically develop steadily throughout the year, with opportunities for practice and links to developing conceptual understanding.

- Much of the learning in Kindergarten—K.CC.C.6, all of K.OA and K.NBT, and K.MD.B.3—depends on the foundational ability to count to answer "how many?" (K.CC.5), which, itself, is grounded in K.CC.B.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Working with numbers 11–19 (K.NBT) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3).
- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.A.3). See below under "Examples of Linking Supporting Clusters to the Major Work of the Grade."
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers; see below under "Examples of Linking Supporting Clusters to the Major Work of the Grade."

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

K.CC.B.5 Cardinal counting should be a focus in itself as needed, and should be a main component of other work in the Kindergarten classroom. Opportunities to develop students' understanding of cardinality

abound, both within the instructional time devoted specifically to mathematics (as noted above) and elsewhere in the instructional day.

K.OA.A.2 Through representing and solving addition and subtraction problems (see also K.OA.A.1), students understand addition as joining and adding to, and understand subtraction as separating and taking from. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time, with subtraction coming to be understood as reversing the actions involved in addition and as finding an unknown addend (see 1.OA.B.4).[39]

K.OA.A.3 Connected with other standards such as K.OA.A.1 and K.OA.A.2, the ability to decompose numbers flexibly is a key focus. At this age, even children who seem competent at counting can hold conversations like this:

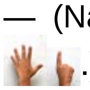
—Happy birthday, Naomi! Tell the class how old you are today.

—I'm *six!* (Naomi shows .)

—Oh, so you are *this* old? (Teacher shows .)

— (Naomi giggles and says.) *No!* I'm *this!* (And she re- shows .)

—Oh, so are you *this* old, right? (Teacher shows .)

— (Naomi giggles again.) *Noooo!* I'm *this old!* (And she again shows .)

Naomi can count, mostly accurately, but seemed to believe there was only one correct way to show 6 and so didn't pursue the matter further!

K.OA.A. 4 **"**Making ten" will become a key strategy (in grade 1) for adding and subtracting within 20; students gain the foundations for this in Kindergarten by finding the number that makes 10 when given another number. Over the course of the year, given playful opportunities (e.g., a "how many fingers *don't* you see" game), many Kindergarten children can become fluent with the pairs of numbers that make 10, and can, when a

[39] pp. 2, 3 of the *Progression* document for K Counting and Cardinality and K-5 Operations and Algebraic Thinking. Available at www.ime.arizona.edu/progressions.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

number less than 10 is named, name the "missing amount" even without looking at fingers.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected to the of the grade. Class discussions and mathematical tasks (short, longer, scaffolded and unscaffolded) are an important opportunity to develop good practice while learning new content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the standards for mathematical practice follow.

Kindergarten students *speak* the number names by ones and by tens all the way to 100 (K.CC.A.1). The structure of a number name like "thirty-two" reflects the underlying system of place value. Attending to and using that structure (MP.7) is, even more than K.NBT.1, an important foundation for place value. Children who can recite "thirty, forty, fifty, sixty,…" can also hear the rhyme in "thirty-eight, forty-eight, fifty-eight, sixty-eight,…" counting by 10s not starting at 0. Hearing this structure prepares them to read those numbers in grade 1. See the *Progression* document for K Counting and Cardinality and K-5 Operations and Algebraic Thinking for more information about how patterns in the number names affect learning (including deviations from those patterns, as in "sixteen" which puts the ones digit first).

- As children learn to count by tens (K.CC.A.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence ten, twenty, thirty… as a new child joins the ones already standing in front of the classroom and showing all their fingers. Connecting *six* children with the spoken *sixty*, and *seven* children with the spoken *seventy*, and *eight* children with the spoken *eighty* also draws attention to structure (MP.7) in a way that prepares for place value.

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Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). The equations that the teacher writes on the board to describe these situations (such as 8 + 2 = 10) are also mathematical models.
- If a student chooses to use objects, fingers, or a math drawing to analyze and solve a word problem, then it is an example of the student using an appropriate tool strategically (MP.5).

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[40] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

[40] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf.

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Counting and Cardinality

- ■ D. Know number names and the count sequence.
- ■ E. Count to tell the number of objects.
- ■ F. Compare numbers

Operations and Algebraic Thinking

- ■ B. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- ■ B. Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- ○ C. Describe and compare measurable attributes.
- □ D. Classify objects and count the number of objects in categories.

Geometry

- ○ C. Identify and describe shapes.
- □ D. Analyze, compare, create, and compose shapes.

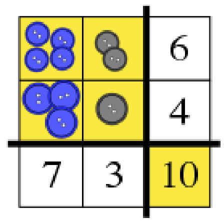
Examples of Linking Supporting Clusters to the Major Work of the Grade

So much is brand new to children in Kindergarten that as much as possible, everything throughout the school day should support everything else, as for example when language supports number.

- Even within mathematics itself, understanding for example that 18 is "ten ones and [eight more] ones" (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose 10 in more than one way (see K.OA.A.3). For example, given a collection of 10 buttons, children could classify by color and size to answer (K.CC.B.5) questions like "how many *small* buttons do you have" or "how many blue buttons do you have" or "how many large gray buttons do you have?" Such a decomposition of objects can show both 10 = 7 + 3 and 10 = 6 + 4. (See figure.)

Mathematics |Grade K

PARCC Model Content Frameworks Indications (continued)



- Students can count vertices (see K.CC.B.5) as a strategy for recognizing shapes in different orientations (see K.G.A.2) and can use shapes as a setting in which to compare numbers (see K.CC.C.6; e.g., count to see which has more vertices, an octagon or a hexagon—see K.G.B.4).

Grade K

Counting and Cardinality (CC)

Know number names and the count sequence		**Major**
K.CC.1	Count to 100 by ones and by tens.	
K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	
K.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	
Count to tell the number of objects		**Major**
K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	
K.CC.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	
Compare numbers		**Major**
K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]	
K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.	

Operations and Algebraic Thinking (OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from		**Major**
K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	
K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	
K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	
K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	
K.OA.5	Fluently add and subtract within 5.	

Grade K

Number and Operations in Base Ten (NBT)

Work with numbers 11-19 to gain foundations for place value		**Major**
K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	

Measurement and Data (MD)

Describe and compare measurable attributes		**Additional**
K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	
K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	
Classify objects and count the number of objects in each category		**Supporting**
K.MD.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]	

Geometry (G)

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)		**Additional**
K.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind*, and *next to.*	
K.G.2	Correctly name shapes regardless of their orientations or overall size.	
K.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	
Analyze, compare, create, and compose shapes		**Supporting**
K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	
K.G.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	
K.G.6	Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	

[1] Include groups with up to ten objects.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

[3] Limit category counts to be less than or equal to 10.

Grade K

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 1

In Grade 1, instruction should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes. Each critical area is described below.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations,**

Grade 1

Geometry, Statistics & Probability (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

[1] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade K to Grade 1

- Grade K students determined sums and differences primarily by representing problems in concrete terms. In grade 1, students gradually come to use more sophisticated strategies (such as making ten) that depend on the properties of addition and subtraction.
- Given the larger numbers studied in grade 1, students must begin to use and understand the symbol system for writing numbers. This includes reading and writing numbers through 120, and understanding the early elements of place value, in particular being able to think of a ten as a unit and understanding that the digits of a two-digit number represent the number of complete tens in that number, and the number of remaining ones.
- Using this understanding of place value and the properties of operations, grade 1 students will represent, explain, and perform addition and subtraction of two-digit numbers in specified cases.
- Students in K learned about the meanings of addition and subtraction as ways of finding what happens when collections are combined or separated, or when their sizes are changed. Building on this base of meaning, grade 1 students represent and solve a large variety of addition and subtraction problems (word problems and problems set in classroom discussions of various scenarios involving putting together, taking apart, comparing, and so on, with different quantities in the problem unknown).[41]

Grade K students generally saw equations only when the teacher wrote them on the board; Kindergarten students were not expected to write equations themselves. Grade 1 students will write equations for a variety of reasons, such as expressing a decomposition of a number (16 = 9 + 7), expressing a piece of reasoning about numbers (9 + 7 = 9 + 1 + 6 along the way to making ten), or representing a word problem with an unknown (9 + ? = 16). Students use the equal sign appropriately, evaluate the truth of an equation, and determine unknown numbers that will make an equation true.

Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Students are fluent in all additions and subtractions within 10. (Note that with sufficient experiences and practice throughout the year, many students will also become fluent with additional additions and subtractions beyond 10, e.g., 10 + 6 = 16, or 17 + 1 = 18.)

[41] See Table 2, p. 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

1.NBT.C.5 Students are fluent at mentally finding 10 more or 10 less than any given two-digit number without having to count. (This standard is best thought of as an indicator of whether students are understanding place value for two-digit numbers.)

The explicit fluency standard 1.OA.C.6 also has implications for fluency in other standards, including:

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. This standard relates to fluency when the additions and subtractions are within 10, as they are in the italicized examples given along with the standard: *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. This standard is closely related to fact families; for example, knowing the fact family for 8 + 3 = 11 means being able to find the unknown number in 8 + ? = 11.

Examples of Major Within-Grade Dependencies

- Standard 1.OA.B.3 calls for students to “apply properties of operations” and gives the example “If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.” Similarly, knowing 13 – 3 gives a good starting place for figuring out 13 – 4. Use of properties lets us apply knowledge we have to situations we need to figure out. But *all* of that depends on having some starting places. Standards 1.OA.C.6 and 1.NBT.C.5 are such starting places, and are essential building blocks for all of the arithmetic of grade 1. They must therefore be given ample attention early in the year. Though often notated on paper, these, and 1.NBT.6, are essentially *mental* arithmetic knowledge and reasoning (though 1.NBTB..6 is still treated as developing, not yet fluent and fully mental).

- 1.NBT.B.2 describes the place-value foundations for 1.NBT.B., and 1.NBT.C.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- A thorough understanding of how place value language and notation (which are mostly consistent with each other in English) represents number (1.NBT.A) serves calculation (1.NBT.B) in many ways—not just pencil and paper calculation, but mental calculation as well. It is valuable for calculation to know that numbers are

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

named so that "twenty-eight" with the "eight" taken away leaves "twenty." That is, the names are designed to make that calculation easy so that we can base harder calculations like 28 – 9 and 28 – 7 on it using properties of the operations 1.OA.3. Similarly "twenty-eight" with the "twenty" taken away leaves "eight," allowing us to base harder calculations on *that*.

The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multi-digit addition and subtraction (1.NBT). For example, when teaching a new situation type such as Compare, the numbers can initially be small enough so that a math drawing shows all of the objects in the problem; this keeps the focus on the situation, and allows students not yet fluent to absorb it. Once the situation type is better understood, and students grow in fluency, larger numbers can begin to appear in such problems. At this point an equation can be used to represent the problem, and fluency and computational skill become involved in finding the answer.[42]

- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, Put Together/Take Apart problems with Addend Unknown can show subtraction as finding an unknown addend.[43]

- Units are a connection between place value (1.NBT) and measurement (1.MD). Grade 1 is when students first encounter the concept of a tens unit, and it is also when they first encounter the concept of a length unit. In later grades, unit thinking will become important throughout arithmetic, including in the development of multi-digit multiplication and division algorithms, and the development of fraction concepts and operations.

- Measurement standards 1.MD.A.1 and 1.MD.A.2, together, support and provide a context for the 1.OA.1 goal of solving subtraction problems that involve comparing. To meet standard 1.MD.A.1, students compare the lengths of two objects by means of a third object. In some cases, that third object might be a length of string that allows a

[42] See p. 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.
[43] See p. 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

"copy" of the length of one immovable object to be moved to another location to compare with the length of another movable object. But that third object might be some kind of unit—the difference between two students' heights measured in inch cubes; or the size of two whales, measured in students stretched end-to-end. When students cannot find the *exact* difference because of the magnitude of the numbers that arise from measurement—as may occur in comparing two students' heights—they may still compare the measurements to know which is greater (1.NBT.3). (Grade 2 standard 2.MD.6 formalizes this idea on a number line diagram.) While children are dealing only with the limited precision of whole and half-hours, they must distinguish the position of the longer hand (the "big hand"). From the top of the circle at the top of the hour (not a common expression in American English) pointing to 12, that hand must go *half* way around the circle over the course of half of an hour. Then another half way around for another half hour—the two halves making a full hour's time. The fact that half way around takes them from 12 to the number that is half of 12 may *not* be a good connection to make—the half hour is not 6 of any standard units, nor is the hour 12 of them—but geometry standard 1.G.A.3, partitioning circles into halves and quarters, does connect well.

- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This is also connected to length measurement (1.MD.A.2), since students must visualize an object to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two 5s into a 10 uses, the idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations is a big idea, common across mathematics.

Examples of Opportunities for In-Depth Focus

1.NBT.B.2 Grade 1 is students' first encounter with the three linked components of the place value system: base ten units, bundling and unbundling of units, and positional notation. Understanding place value is the foundation of the entire NBT domain.

1.NBT.C.4 Understanding place value is not a final goal on its own; the goal is to use place value understanding and properties of operations to add and

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

subtract (1.NBT.B). Students learn how standard notation presents and/or records problems for which students have developed mental strategies—adding 10 (1.NBTC..5) repeatedly and adding 1 repeatedly (counting on, 1.OA.C.6) to a two-digit number—and extends that to adding two arbitrary two-digit numbers (with a result within 100). Being able to represent these additions with materials that show the base 10 structure, having a strong mental image and ability with simple versions of these additions, and understanding how the notation records these additions, and being able to interpret subtraction in its relation to addition are the foundation for all future arithmetic in elementary school.

1.OA.A.1 There are many distinct elementary addition and subtraction situations; students in grade 1 should work extensively with all of them. (The harder situation types need not be mastered until grade 2.)[44]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected to the of the grade. Class discussions and mathematical tasks (short, longer, scaffolded and unscaffolded) are an important opportunity to develop good practice while learning new content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the standards for mathematical practice follow.

- All work with properties (1.OA.B.3) and with understanding and using place value (e.g., 1.NBT.B.2, 1.NBT.C.4) should be seen as an investigation and use of the structure of our number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts (1.NBT.C.4, 1.NBT.C.5, 1.NBT.C.6) are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief, but excellent arguments at this grade level could include:
    - I know that 7 – 3 equals 4 because 4 + 3 equals 7 (shows 1.OA.4 being met).
    - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
    - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.

[44] See Table 2, p. 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

- The experience of starting at some number (e.g., 23) and *counting* on 10, and then 10 more, and then 10 more, and then 10 more, and so on, and *hearing* the repetition "thirty-three, forty-three, fifty-three, sixty-three) is often a bit of a surprise to children, and quite powerful. From the repeated reasoning, they abstract a pattern (MP.8): they describe this
  in various ways, but sometimes say variations on "adding 10 to any number rhymes" and it "changes the counting-by-tens word I use." Thus, content standard 1.NBT.4 is being approached by applying standard MP.8 of mathematical practice.

- Students in grade 1 work with some sophisticated addition and subtraction situations (1.OA.1), such as "Julie has 8 more apples than Lucy. Julie has 12 apples. How many apples does Lucy have?" The equations 12 – 8 = ? and ? + 8 = 12 are both mathematical models of this situation (MP.4).

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[45] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[45] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

Mathematics | Grade 1

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	E. Represent and solve problems involving addition and subtraction
■	F. Understand and apply properties of operations and the relationship between addition and subtraction.
■	G. Add and subtract within 20.
■	H. Work with addition and subtraction equations.
Number and Operations in Base Ten	
■	D. Extend the counting sequence.
■	E. Understand place value.
■	F. Use place value understanding and properties of operations to add and subtract.
Measurement and Data	
■	D. Measure lengths indirectly and by iterating length units.
○	E. Tell and write time.
□	F. Represent and interpret data.
Geometry	
○	B. Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

When students work with organizing, representing, and interpreting data, the process include practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.4 after the semicolon, and see the K-5 MD Progression document, especially Table 1 on page 4 and the discussion of categorical data on pp. 5, 6).[46]

Telling and writing time on digital clocks (1.MD.3) is a context in which one can practice reading numbers (1.NBT.1), a kind of "application," but no more. Relating those times to *meanings*—events during a day—is not part of the 1.MD.3 *content* standard, but making sense of what one is doing (MP.1), and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

[46] http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

Grade 1

Operations and Algebraic Thinking (OA)

Represent and solve problems involving addition and subtraction		**Major**
1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]	
1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	
Understand and apply properties of operations and the relationship between addition and subtraction		**Major**
1.OA.3	Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*	
1.OA.4	Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	
Add and subtract within 20		**Major**
1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	
1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	
Work with addition and subtraction equations		**Major**
1.OA.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	
1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*	

Grade 1

Number and Operations in Base Ten (NBT)

Extend the counting sequence		**Major**
1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	
Understand place value		**Major**
1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	
Use place value understanding and properties of operations to add and subtract		**Major**
1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	
1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	
1.NBT.6	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	

Measurement and Data (MD)

Measure lengths indirectly and by iterating length units		**Major**
1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	
1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps*.	

Grade 1

Tell and write time		**Additional**
1.MD.3	Tell and write time in hours and half-hours using analog and digital clocks.	
Represent and interpret data		**Supporting**
1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	
Geometry (G)		
Reason with shapes and their attributes		**Additional**
1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	
1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]	
1.G.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	

[2] See Glossary, Table 1.

[3] Students need not use formal terms for these properties.

[4] Students do not need to learn formal names such as "right rectangular prism."

Grade 1

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

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Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 2

In Grade 2, instruction should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Each critical area is described below.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 2

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 1 to Grade 2

- Where Grade 1 students worked within 100, Grade 2 students will read and write numbers through 1000, extending their understanding of place value to include units of hundreds.
- Similarly, Grade 2 students use their understanding of place value to add and subtract within 1000 (e.g., 237 + 616 or 822 – 237). They can explain what they are doing as they add and subtract, and they become fluent in the special case of addition and subtraction within 100.
- For situation-based problems (e.g., word problems), students extend their ability by solving two-step problems using addition, subtraction, or both operations. They also master the harder kinds of one-step addition and subtraction problems in this grade (such as Take From with Start Unknown).[47]
- Students use standard units of measure and appropriate measurement tools. They understand basic properties of linear (length/distance) measurement, such as the fact that the smaller the unit, the more iterations will be needed to cover a given length.

Fluency Expectations or Examples of Culminating Standards

2.OA.B.2 Fluently add and subtract within 20 mentally. By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (Students can also show their fluency using an efficient, general algorithm.)[48]

Note: The following standards don't explicitly state a fluency requirement, but fluency is also important in these cases for the reasons stated.

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, 100s. (A lack of fluency here can signal a lack of understanding. Skip counting is also sometimes a strategy for adding or subtracting, so fluency is helpful.)

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (Students who struggle to read a three-digit number may not grasp place value.)

2.NBT.B.8 Given any number between 100 and 900, mentally add or subtract 10 or 100. (A lack of fluency here can signal a lack of understanding.)

[47] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

[48] For the difference between a computation *strategy* and a computation *algorithm*, see the Glossary, page 330).

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (Sufficient practice is required in order to measure accurately and reasonably quickly.)

Students' ability to *explain* (2.NBT.B.9) what they know in a fluent and clear way will continue to develop for years, but the *understanding* reflected in that standard should be a major focus begun relatively early in grade 2 and should be considered a "culminating standard" at the end of the year, an understanding that can be *assumed* in later grades.

Examples of Major Within-Grade Dependencies

- Cluster 2.NBT.A—understanding place value—is the foundation for cluster 2.NBT.B, using place value understanding and the properties of operations to add and subtract. (This does not mean that 2.NBT.A must be entirely "completed" before beginning work with 2.NBT.B; mastery of the two clusters can grow over time in tandem with one another.)
- Knowing single-digit sums from memory (2.OA.B.2) is required for adding and subtracting multi-digit numbers fluently and efficiently (2.NBT.B).
- Cluster 2.NBT.B in turn is an essential foundation for *all* the major arithmetic work of grade 2 and beyond.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Representing whole numbers as lengths (2.MD.B.6) and comparing measurements (2.MD.A.4) can build a robust and flexible model for fluent subtraction (2.OA.A.1). For example, a good way to see the "distance" from 6 to 20 is to see the distance from 6 to 10 joined with the distance from 10 to 20. (See *Complements* in "Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice," below.)
- Problems involving dollars, dimes, and pennies (2.MD.C.8) should be connected with the place value learning of 100s, 10s, and 1s (2.NBT.A.1). Though the notation is different, a dollar is 100 cents, a "bundle" of ten dimes, each of which is a "bundle" of ten pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes), and additions that are appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is *not* a standard at this grade).

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PARCC Model Content Frameworks Indications (continued)

- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multi-digit addition and subtraction (2.OA.B.2, 2.NBT.B).[49]

- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with groups of 5 (2.OA.C.4). (See also *Knowing 5* in "Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice," below.) Recognizing time by seeing the minute hand at 3 and *knowing* that is fifteen minutes; recognizing three nickels as 15¢; and seeing the 15-ness of a 3-by-5 rectangular array held in any position at all (including with neither base horizontal) will prepare for understanding what the new operation of multiplication *means*, and not just the particular facts that it uses.

Examples of Opportunities for In-Depth Focus

2.OA.A.1 Using situations (from word problems, from classroom events or experiences, and from discovered mathematical patterns) as a source of problems can help students make sense of and contextualize the operations they are learning. There is a tremendous variety of basic situation types in addition and subtraction.[50]

2.NBT.B.7 It takes substantial time throughout the year for students to extend addition and subtraction to 1000, connecting steps in the computation to what they know about place value and properties of operations.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Fidelity to mathematical practice should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to develop good practice while learning new

[12] Easier word problem types, such as Result Unknown, should generally involve more difficult sums and differences. The challenge in these problems is not algebraic, but rather computational, and the problems can build computational skill in context. Harder situation types, such as Start Unknown, or Compare, should sometimes involve easy computations and sometimes involve harder computations. (Having harder computations with a harder situation type may be counterintuitive, but if the harder situation types always involve easier numbers, then students will never have need or incentive to represent them with equations, when means they won't have opportunities to connect the structure of the representation to the structure of the problem.)

[50] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf as well as www.achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

content and to learn content through proper practice. Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow. In all grades, there is plentiful opportunity for curriculum and instruction to meet the content standards with fidelity to mathematical practice. Standard MP.1 should be part of the life of the classroom at all grades and at most times. The standards for using and expressing the regularity in repeated reasoning (MP.8) and looking for and making use of structure (MP.7) are especially well suited to two major content areas of Grade 2, and might be the principal aspects of practice to take advantage of at this grade level.

- *Complements:* Given the opportunity, children fairly readily generalize the idea of partners of 10, and they feel proud of being able to name pairs, say, to 30 (e.g., knowing that 12 and 18 are ‘partners of 30’). After practice pairing only multiples of 10, they can name pairs to 100. This approach—learning a set of “content” facts and recognizing/building a structure behind them (MP.7) through repeated use in varied contexts (MP.8)—makes very appropriate use of good mathematical practice.
- *Doubling and Halving:* Seeing even numbers as the sum of two equal addends (“doubling”) and seeing the number of pairs contained within an even number (“halving”) is an extremely powerful foundation for multiplication and its central property (the distributive property) and for later study of fractions. If doubling is not restricted just to numbers less than 20, but extended to include two-digit numbers with digits less than 5, students can use physical and, not long after, *mental* images of dimes and pennies, or base ten rods and units, to double numbers like “four hundred” to “eight hundred” (spoken) easily as they double 4 to 8, and later to double 40 to 80 the same way, and still later hold enough in their heads to double 32 and 24 (with no more mathematical knowledge, but more increase in working memory and executive function to track the process). Likewise, they can halve two-digit numbers both of whose digits are even by picturing these as composed of dimes and pennies (or some other material that distinguishes 10s and 1s). The *practice* is just with basic facts that are part of grade 2 standards, and the varied contexts helps make these secure and also prepare the ground for later learning, so that the distributive property of multiplication in Grade 3 is not a brand new idea but has already become a familiar structure (MP.7) with an experience-based (MP.8) and intuitive underpinning. In the same way, halving becomes a robust model for other fractions.

Although children in Grades K-1 have used objects and drawings as appropriate tools to represent mathematical ideas and operations, Grade 2 is the first time that students make more than passing use of objects that are not just school-artifacts, like blocks and

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

rods, but remain appropriate tools (MP.5) for a lifetime: rulers, clocks, coins, and the number line (essentially an abstract ruler or measurement scale).

- *Knowing 5:* Standard MP.5 is about not just the ability to *use* tools, but the ability to *choose* the appropriate tool for a task. At this stage, because students are just beginning to use a variety of tools, their utility may seem both obvious and fixed to a task: a ruler measures length; a clock measures time; a coin "measures" money, and children have little to make choices about. But coming to understand the *significance* of counting by 5—the usefulness of that litany and the situations in which it appears—may involve, for some children, a choice of which of several images (nickels, hands, telling time) is most clarifying and salient to them. Generating the abstraction—the litany 0, 5, 10, 15, etc.—may also be aided by experiences in the various domains, the recognition that one sequence of numbers is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.

A note on manipulatives in grades K-2: Manipulatives such as physical models of hundreds, tens, and ones are an important part of the K-2 classroom. These manipulatives should always be connected to written symbols and methods.[51] Also, in practical terms, an important design principle is that when distributing manipulatives, it is helpful to do a lot of related work with them to keep interest and because handing them out can take a lot of time.[52]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

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[51] See p. 19 of the K-8 Publishers' Criteria, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[52] Reference unknown.

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ▣ Supporting Clusters; ⬡ Additional Clusters

Operations and Algebraic Thinking	
■	D. Represent and solve problems involving addition and subtraction
■	E. Add and subtract within 20.
▣	F. Work with equal groups of objects to gain foundations for multiplication.
Number and Operations in Base Ten	
■	C. Understand place value.
■	D. Use place value understanding and properties of operations to add and subtract.
Measurement and Data	
■	E. Measure and estimate lengths in standard units.
■	F. Relate addition and subtraction to length.
▣	G. Work with time and money.
▣	H. Represent and interpret data.
Geometry	
⬡	B. Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with time and money (2.MD.C), their work with dollars, dimes, and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.2), and with arrays of five rows and/or five columns (2.OA.C) should be taken together.
- In cluster 2.MD.D, "Represent and interpret data," it is particularly standard 2.MD.10 that represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can be a visually appealing context for solving addition and subtraction problems. The language in 2.MD.10 mentions word problems (2.OA) explicitly. See the *Progression* document for K-5 Measurement and Data for more on the connections between data work and arithmetic in the early grades.[53]

[53] http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

Mathematics | Grade 2

PARCC Model Content Frameworks Indications (continued)

- Without adding greater meaning or depth, 2.MD.9 is a potential context for 2.MD.1 and gives students a first taste of visual comparison of numerical information (though the fact that this numerical information was derived from length makes the representation more about scaling the information than about visualizing it).

Grade 2

Operations and Algebraic Thinking (OA)		
Represent and solve problems involving addition and subtraction		**Major**
2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	
Add and subtract within 20		**Major**
2.OA.2	Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.	
Work with equal groups of objects to gain foundations for multiplication		**Supporting**
2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	
Number and Operations in Base Ten (NBT)		
Understand place value		**Major**
2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	
2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.	
2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	
2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	
Use place value understanding and properties of operations to add and subtract		**Major**
2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	
2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	

Grade 2

2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

Measurement and Data (MD)

Measure and estimate lengths in standard units		**Major**
2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.	
2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	
Relate addition and subtraction to length		**Major**
2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	
2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	
Work with time and money		**Supporting**
2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	
2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	
Represent and interpret data		**Supporting**
2.MD.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	
2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.	

Grade 2

Geometry (G)

Reason with shapes and their attributes		Additional
2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	
2.G.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	

[1] See Glossary, Table 1.

[2] See standard 1.OA.6 for a list of mental strategies.

[3] Explanations may be supported by drawings or objects.

[4] See Glossary, Table 1.

[5] Sizes are compared directly or visually, not compared by measuring.

Grade 2

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 3

In Grade 3, instruction should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Each critical area is described below.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 2 to Grade 3

- Students in grade 3 begin to enlarge their concept of number by developing an understanding of fractions as numbers. This work will continue in grades 3-6, preparing the way for work with the complete rational number system in grades 6 and 7.

- Students in grades K-2 worked on number; place value; and addition and subtraction concepts, skills and problem solving. Beginning in grade 3, students will learn concepts, skills, and problem solving for multiplication and division. This work will continue in grades 3, 4 and 5, preparing the way for work with ratios and proportions in grades 6 and 7.

Fluency Expectations or Examples of Culminating Standards

- **3.OA.C.7** Students fluently multiply and divide within 100. By the end of grade 3, they know all products of two one-digit numbers from memory.

- **3.NBT.A.2** Students fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (Although **3.OA.C.7** and **3.NBT.A.2** are both fluency standards, these two standards do not represent equal investments of time in grade 3. Note that students in grade 2 were already adding and subtracting within 1000, just not fluently. That makes **3.NBT.2** a relatively small and incremental expectation. By contrast, multiplication and division are new in grade 3, and meeting the multiplication and division fluency standard **3.OA.C.7** with understanding is a major portion of students' work in grade 3.)

Examples of Major Within-Grade Dependencies

- Students must begin work with multiplication and division (**3.OA**) at or near the very start of the year to allow time for understanding and fluency to develop. Note that area models for products are an important part of this process (**3.MD.C.7**). Hence, work on concepts of area (**3.MD.C.5–6**) should likely begin at or near the start of the year as well.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with partitioning shapes (**3.G.A.2**) relates to visual fraction models (**3.NF**).

- Scaled picture graphs and scaled bar graphs (**3.MD.B.3**) can be a visually appealing context for solving multiplication and division problems.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

3.OA.A.3	Word problems involving equal groups, arrays, and measurement quantities can be used to build students' understanding of and skill with multiplication and division, as well as to allow students to demonstrate their understanding of and skill with these operations.
3.OA.C.7	Finding single-digit products and related quotients is a required fluency for grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply, and divide with fractions. After multiplication and division situations have been established, reasoning about patterns in products (e.g., products involving factors of 5 or 9) can help students remember particular products and quotients. Practice — and if necessary, extra support — should continue all year for those who need it to attain fluency.
3.NF.A.2	Developing an understanding of fractions as numbers is essential for future work with the number system. It is critical that students at this grade are able to place fractions on a number line diagram and understand them as a related component of their ever-expanding number system.
3.MD.A.2	Continuous measurement quantities such as liquid volume, mass, and so on are an important context for fraction arithmetic (cf. **4.NF.B.4c, 5.NF.B.7c, 5.NF.B.3**). In grade 3, students begin to get a feel for continuous measurement quantities and solve whole-number problems involving such quantities.
3.MD.C.7	Area is a major concept within measurement, and area models must function as a support for multiplicative reasoning in grade 3 and beyond.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

- Students learn and use strategies for finding products and quotients that are based on the properties of operations; for example, to find 4×7, they may recognize that $7 = 5 + 2$ and compute $4 \times 5 + 4 \times 2$. This is an example of seeing and making use of structure (**MP.7**). Such reasoning processes amount to brief arguments that students may construct and critique (**MP.3**).
- Students will analyze a number of situation types for multiplication and division, including arrays and measurement contexts. Extending their understanding of multiplication and division to these situations requires that they make sense of problems and persevere in solving them (**MP.1**), look for and make use of structure (**MP.7**) as they model these situations with mathematical forms (**MP.4**), and attend to precision (**MP.6**) as they distinguish different kinds of situations over time (**MP.8**).

Content Emphases by Cluster[54]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

[54] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

■ E. Represent and solve problems involving multiplication and division.

■ F. Understand properties of multiplication and the relationship between multiplication and division.

■ G. Multiply and divide within 100.

■ H. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

○ B. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions

■ B. Develop understanding of fractions as numbers.

Measurement and Data

■ E. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

◘ F. Represent and interpret data.

■ G. Geometric measurement: understand concepts of area and relate area to multiplication and addition.

○ H. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

◘ B. Reason with shapes and their attributes.

Mathematics | Grade 3

PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (**3.MD.B.3**). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.
- Reason with shapes and their attributes: Work toward meeting **3.G.A.2** should be positioned in support of area measurement and understanding of fractions.

Grade 3

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division		**Major**
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	
3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*	
Understand properties of multiplication and the relationship between multiplication and division		**Major**
3.OA.5	Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	
3.OA.6	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	
Multiply and divide within 100		**Major**
3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	
Solve problems involving the four operations, and identify and explain patterns in arithmetic		**Major**
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]	
3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	

Grade 3

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic [4]	**Additional**

3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions [5] (NF)

Develop understanding of fractions as numbers	**Major**

3.NF.1	Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data (MD)

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects	**Major**

3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Grade 3

3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]	
Represent and interpret data		**Supporting**
3.MD.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	
Geometric measurement: understand concepts of area and relate area to multiplication and to addition		**Major**
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	
3.MD.6	Measure areas by counting unit squares (square cm, square *m*, square *in*, square *ft*, and improvised units).	
3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures		**Additional**
3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	

Grade 3

Geometry (G)

Reason with shapes and their attributes		Supporting
3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	
3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*.	

[1] See Glossary, Table 2.

[2] Students need not use formal terms for these properties.

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

[4] A range of algorithms may be used.

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

[6] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7] Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Grade 3

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 4

In Grade 4, instruction should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Each critical area is described below.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and

Grade 4

conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 3 to Grade 4

- In grade 3, students studied multiplication in terms of equal groups, arrays and area. In grade 4, students extend their concept of multiplication to make multiplicative comparisons (4.OA.A.1).[55]

- Students in grade 4 apply and extend their understanding of the meanings and properties of addition and subtraction of whole numbers to extend addition and subtraction to fractions (4.NF.B.3).[56]

- Fraction equivalence is an important theme within the standards that begins in grade 3. In grade 4, students extend their understanding of fraction equivalence to the general case, $a/b = (n \times a)/(n \times b)$ (3.NF.A.3 → 4.NF.A.1).[57] They apply this understanding to compare fractions in the general case (3.NF.A.3d → 4.NF.A.2).

- Students in grade 4 apply and extend their understanding of the meanings and properties of multiplication of whole numbers to multiply a fraction by a whole number (4.NF.B.4).

- Students in grade 4 begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass and time (4.MD.A.2).

- Students combine their understanding of the meanings and properties of multiplication and division with their understanding of base-ten units to begin to multiply and divide multidigit numbers (4.NBT.B.5–6; this builds on work done in grade 3, cf. 3.NBT.A.3).

- Students generalize their previous understanding of place value for multidigit whole numbers (4.NBT.A.1–3). This supports their work in multidigit multiplication and division, carrying forward into grade 5, when students will extend place value to decimals.

Fluency Expectations or Examples of Culminating Standards

4.NBT.B.4 Students fluently add and subtract multidigit whole numbers using the standard algorithm.

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[55] In an additive comparison problem (grades 1–2), the underlying question is *what amount would be added to one quantity to result in the other?* In a multiplicative comparison problem, the underlying question is *what factor would multiply one quantity to result in the other?*
[56] This work is limited to equal denominators in grade 4 to give students more time to build their understanding of fraction equivalence, before adding and subtracting unlike denominators in grade 5.
[57] Students who can generate equivalent fractions can also develop strategies for adding fractions with different denominators, but this is not a requirement in grade 4.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

Examples of Major Within-Grade Dependencies

- Students' work with decimals (4.NF.C.5–7) depends to some extent on concepts of fraction equivalence and elements of fraction arithmetic. Students express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100; comparisons of decimals require that students use similar reasoning to comparisons with fractions.
- Standard 4.MD.A.2 refers to using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, time, and so on. Some parts of this standard could be met earlier in the year (such as using whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit — see also 4.MD.A.1), while others might be met only by the end of the year (such as word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number — see also 4.NF.B.3d and 4.NF.B.4c).
- Standard 4.MD.C.7 refers to word problems involving unknown angle measures. Before this standard can be met, students must understand concepts of angle measure (4.MD.C.5) and, presumably, gain some experience measuring angles (4.MD.C.6). Before that can happen, students must have some familiarity with the geometric terms that are used to define angles as geometric shapes (4.G.A.1).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do with units of measure (4.MD.A.1–2) and with multiplication of a fraction by a whole number (4.NF.B.4) can be connected to the idea of "times as much" in multiplication (4.OA.A.1).
- Addition of fractions (4.NF.B.3) and concepts of angle measure (4.MD.C.5a and 4.MD.C.7) are connected in that a one-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360.

Examples of Opportunities for In-Depth Focus

4.NBT.B.5 When students work toward meeting this standard, they combine prior understanding of multiplication with deepening understanding of the base-ten system of units to express the product of two multidigit numbers as another multidigit number. This work will continue in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NBT.B.6 When students work toward meeting this standard, they combine prior understanding of multiplication and division with deepening understanding of the base-ten system of units to find whole-number

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

quotients and remainders with up to four-digit dividends and one-digit divisors. This work will develop further in grade 5 and culminate in fluency with the standard algorithms in grade 6.

4.NF.A.1	Extending fraction equivalence to the general case is necessary to extend arithmetic from whole numbers to fractions and decimals.
4.NF.B.3	This standard represents an important step in the multi-grade progression for addition and subtraction of fractions. Students extend their prior understanding of addition and subtraction to add and subtract fractions with like denominators by thinking of adding or subtracting so many unit fractions.
4.NF.B.4	This standard represents an important step in the multi-grade progression for multiplication and division of fractions. Students extend their developing understanding of multiplication to multiply a fraction by a whole number.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students decompose numbers into sums of multiples of base-ten units to multiply them (4.NBT.B.5), they are seeing and making use of structure (MP.7). As they illustrate and explain the calculation by using physical or drawn models, they are modeling (MP.4), using appropriate drawn tools strategically (MP.5) and attending to precision (MP.6) as they use base-ten units in the appropriate places.

- To compute and interpret remainders in word problems (4.OA.A.3), students must reason abstractly and quantitatively (MP.2), make sense of problems (MP.1), and look for and express regularity in repeated reasoning (MP.8) as they search for the structure (MP.7) in problems with similar interpretations of remainders.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

Content Emphases by Cluster[58]

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In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

■ D. Use the four operations with whole numbers to solve problems.

◘ E. Gain familiarity with factors and multiples.

○ F. Generate and analyze patterns.

Number and Operations in Base Ten

■ C. Generalize place value understanding for multi-digit whole numbers.

■ D. Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions

■ D. Extend understanding of fraction equivalence and ordering.

■ E. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

■ F. Understand decimal notation for fractions, and compare decimal fractions.

[58] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 4

PARCC Model Content Frameworks Indications (continued)

Measurement and Data

- D. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- E. Represent and interpret data.
- F. Geometric measurement: understand concepts of angle and measure angles.

Geometry

- B. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Gain familiarity with factors and multiples: Work in this cluster supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
- Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

Grade 4

Operations and Algebraic Thinking (OA)

Use the four operations with whole numbers to solve problems		**Major**
4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]	
4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Gain familiarity with factors and multiples		**Supporting**
4.OA.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	
Generate and analyze patterns		**Additional**
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	

Number and Operations in Base Ten [2] (NBT)

Generalize place value understanding for multi-digit whole numbers		**Major**
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	
4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.	

Grade 4

Use place value understanding and properties of operations to perform multi-digit arithmetic		**Major**
4.NBT.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	
4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	

Number and Operations—Fractions[3] (NF)

Extend understanding of fraction equivalence and ordering		**Major**
4.NF.1	Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	
4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers		**Major**
4.NF.3	Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	

Grade 4

4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	
Understand decimal notation for fractions, and compare decimal fractions		**Major**
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*	
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.	
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
Measurement and Data (MD)		
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit		**Supporting**
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),…*	
4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
4.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	

Grade 4

Represent and interpret data		**Supporting**
4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	
Geometric measurement: understand concepts of angle and measure angles		**Additional**
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	
Geometry (G)		
Draw and identify lines and angles, and classify shapes by properties of their lines and angles		**Additional**
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	

[1] See Glossary, Table 2.

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal l to 1,000,000.

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Grade 4

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

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PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 5

In Grade 5, instruction should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume. Each critical area is described below.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations,**

Grade 5

Geometry, Statistics & Probability (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 4 to Grade 5

- In grade 5, students will integrate decimal fractions more fully into the place value system (5.NBT.A.1–4). By thinking about decimals as sums of multiples of base-ten units, students begin to extend algorithms for multi-digit operations to decimals (5.NBT.B.7).

- Students use their understanding of fraction equivalence and their skill in generating equivalent fractions as a strategy to add and subtract fractions, including fractions with unlike denominators.

- Students apply and extend their previous understanding of multiplication to multiply a fraction or whole number by a fraction (5.NF.B.4). They also learn the relationship between fractions and division, allowing them to divide any whole number by any nonzero whole number and express the answer in the form of a fraction or mixed number (5.NF.B.3). And they apply and extend their previous understanding of multiplication and division to divide a unit fraction by a whole number or a whole number by a unit fraction.[59]

- Students extend their grade 4 work in finding whole-number quotients and remainders to the case of two-digit divisors (5.NBT.B.6).

- Students continue their work in geometric measurement by working with volume as an attribute of solid figures and as a measurement quantity (5.MD.C.3–5).

- Students build on their previous work with number lines to use two perpendicular number lines to define a coordinate system (5.G.A.1–2).

Fluency Expectations or Examples of Culminating Standards

5.NBT.B.5 Students fluently multiply multidigit whole numbers using the standard algorithm.

Examples of Major Within-Grade Dependencies

- Understanding that in a multidigit number, a digit in one place represents $^1/_{10}$ of what it represents in the place to its left (5.NBT.A.1) is an example of multiplying a quantity by a fraction (5.NF.B.4).

[59] Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But the division of a fraction by a fraction is not a requirement in this grade.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connections among Standards, Clusters or Domains

- The work that students do in multiplying fractions extends their understanding of the operation of multiplication. For example, to multiply $a/b \times q$ (where q is a whole number or a fraction), students can interpret $a/b \times q$ as meaning a parts of a partition of q into b equal parts (5.NF.B.4a). This interpretation of the product leads to a product that is less than, equal to or greater than q depending on whether $a/b < 1$, $a/b = 1$ or $a/b > 1$, respectively (5.NF.B.5).

- Conversions within the metric system represent an important practical application of the place value system. Students' work with these units (5.MD.A.1) can be connected to their work with place value (5.NBT.A.1).

Examples of Opportunities for In-Depth Focus

5.NBT.A.1 The extension of the place value system from whole numbers to decimals is a major intellectual accomplishment involving understanding and skill with base-ten units and fractions.

5.NBT.B.6 The extension from one-digit divisors to two-digit divisors requires care. This is a major milestone along the way to reaching fluency with the standard algorithm in grade 6 (6.NS.B.2).

5.NF.A.2 When students meet this standard, they bring together the threads of fraction equivalence (grades 3–5) and addition and subtraction (grades K–4) to fully extend addition and subtraction to fractions.

5.NF.B.4 When students meet this standard, they fully extend multiplication to fractions, making division of fractions in grade 6 (6.NS.A.1) a near target.

5.MD.C.5 Students work with volume as an attribute of a solid figure and as a measurement quantity. Students also relate volume to multiplication and addition. This work begins a progression leading to valuable skills in geometric measurement in middle school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students break divisors and dividends into sums of multiples of base-ten units (5.NBT.B.6), they are seeing and making use of structure (MP.7) and attending to precision (MP.6). Initially for most students, multidigit division problems take time and effort, so they also require perseverance (MP.1) and looking for and expressing regularity in repeated reasoning (MP.8).
- When students explain patterns in the number of zeros of the product when multiplying a number by powers of 10 (5.NBT.A.2), they have an opportunity to look for and express regularity in repeated reasoning (MP.8). When they use these patterns in division, they are making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).
- When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (5.MD.C.5), they also have an opportunity to look for and express regularity in repeated reasoning (MP.8). They attend to precision (MP.6) as they use correct length or volume units, and they use appropriate tools strategically (MP.5) as they understand or make drawings to show these units.

Content Emphases by Cluster[60]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

[60] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 5

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

- ○ C. Write and interpret numerical expressions.
- ○ D. Analyze patterns and relationships.

Number and Operations in Base Ten

- ■ C. Understand the place value system.
- ■ D. Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations — Fractions

- ■ C. Use equivalent fractions as a strategy to add and subtract fractions
- ■ D. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- ◘ D. Convert like measurement units within a given measurement system.
- ◘ E. Represent and interpret data.
- ■ F. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- ○ C. Graph points on the coordinate plane to solve real-world and mathematical problems.
- ○ D. Classify two-dimensional figures into categories based on their properties.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Convert like measurement units within a given measurement system: Work in these standards supports computation with decimals. For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.
- Represent and interpret data: The standard in this cluster provides an opportunity for solving real-world problems with operations on fractions, connecting directly to both Number and Operations — Fractions clusters.

Grade 5

Operations and Algebraic Thinking (OA)

Write and interpret numerical expressions		**Additional**
5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	
5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	
Analyze patterns and relationships		**Additional**
5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	

Number and Operations in Base Ten (NBT)

Understand the place value system		**Major**
5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	
5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
5.NBT.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
5.NBT.4	Use place value understanding to round decimals to any place.	
Perform operations with multi-digit whole numbers and with decimals to hundredths		**Major**
5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	
5.NBT.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	

Grade 5

5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	
Number and Operations—Fractions (NF)		
	Use equivalent fractions as a strategy to add and subtract fractions	**Major**
5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	
5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	
	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	**Major**
5.NF.3	Interpret a fraction as division of the numerator by the denominator (*a/b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (*a/b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b* = (*n* × *a*)/(*n* × *b*) to the effect of multiplying *a/b* by 1.	
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	

Grade 5

5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data (MD)

Convert like measurement units within a given measurement system	**Supporting**
5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Represent and interpret data	**Supporting**
5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	**Major**
5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. c. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. d. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Grade 5

<table>
<tr><td>5.MD.5</td><td colspan="2">Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</td></tr>
<tr><td colspan="3">Geometry (G)</td></tr>
<tr><td colspan="2">Graph points on the coordinate plane to solve real-world and mathematical problems</td><td>Additional</td></tr>
<tr><td>5.G.1</td><td colspan="2">Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</td></tr>
<tr><td>5.G.2</td><td colspan="2">Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</td></tr>
<tr><td colspan="2">Classify two-dimensional figures into categories based on their properties</td><td>Additional</td></tr>
<tr><td>5.G.3</td><td colspan="2">Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</td></tr>
<tr><td>5.G.4</td><td colspan="2">Classify two-dimensional figures in a hierarchy based on properties.</td></tr>
</table>

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Grade 5

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

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10. Reason abstractly and quantitatively.
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12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

College- and Career- Readiness Standards for Mathematics (Grades 6-8)

Mathematics | Grade 6

In Grade 6, instruction should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. Each critical area is described below.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for

Mathematics | Grade 6 (continued)

summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 5 to Grade 6

- Students' prior understanding of and skill with multiplication, division, and fractions contribute to their study of ratios, proportional relationships and unit rates (6.RP).
- Students begin using properties of operations systematically to work with variables, variable expressions, and equations (6.EE).
- Students extend their work with the system of rational numbers to include using positive and negative numbers to describe quantities (6.NS.C.5), extending the number line and coordinate plane to represent rational numbers and ordered pairs (6.NS.C.6), and understanding ordering and absolute value of rational numbers (6.NS.C.7).
- Having worked with measurement data in previous grades, students begin to develop notions of statistical variability, summarizing and describing distributions (6.SP).

Fluency Expectations or Examples of Culminating Standards

6.NS.B.2 Students fluently divide multi-digit numbers using the standard algorithm. This is the culminating standard for several years' worth of work with division of whole numbers.

6.NS.B.3 Students fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. This is the culminating standard for several years' worth of work relating to the domains of Number and Operations in Base Ten, Operations and Algebraic Thinking, and Number and Operations — Fractions.

6.NS.A.1 Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. This completes the extension of operations to fractions.

Examples of Major Within-Grade Dependencies

- Equations of the form $px = q$ (6.EE.B.7) are unknown-factor problems; the solution will sometimes be the quotient of a fraction by a fraction (6.NS.A.1).
- Solving problems by writing and solving equations (6.EE.B.7) involves not only an appreciation of how variables are used (6.EE.B.6) and what it means to solve an equation (6.EE.B.5) but also some ability to write, read, and evaluate expressions in which letters stand for numbers (6.EE.A.2).

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

- Students must be able to place rational numbers on a number line (6.NS.C.7) before they can place ordered pairs of rational numbers on a coordinate plane (6.NS.C.8). The former standard about ordering rational numbers is much more fundamental.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with ratios and proportional relationships (6.RP) can be combined with their work in representing quantitative relationships between dependent and independent variables (6.EE.C.9).
- Plotting rational numbers in the coordinate plane (6.NS.C.8) is part of analyzing proportional relationships (6.RP.A.3a, 7.RP.A.2) and will become important for studying linear equations (8.EE.C.8) and graphs of functions (8.F).[61]
- Students use their skill in recognizing common factors (6.NS.B.4) to rewrite expressions (6.EE.A.3).
- Writing, reading, evaluating, and transforming variable expressions (6.EE.A.1–4) and solving equations and inequalities (6.EE.B.7–8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (6.G.A.2).
- Working with data sets can connect to estimation and mental computation. For example, in a situation where there are 20 different numbers that are all between 8 and 10, one might quickly estimate the sum of the numbers as $9 \times 20 = 180$.

Examples of Opportunities for In-Depth Focus

6.RP.A.3 When students work toward meeting this standard, they use a range of reasoning and representations to analyze proportional relationships.

6.NS.A.1 This is a culminating standard for extending multiplication and division to fractions.

6.NS.C.8 When students work with rational numbers in the coordinate plane to solve problems, they combine and consolidate elements from the other standards in this cluster.

6.EE.A.3 By applying properties of operations to generate equivalent expressions, students use properties of operations that they are familiar with from

[61] While not required by the standards, it might be considered valuable to expose students to time series data and to time graphs as an appealing way to work with rational numbers in the coordinate plane (6.NS.C.8). For example, students could create time graphs of temperature measured each hour over a 24-hour period in a place where, to ensure a strong connection to rational numbers, temperature values might cross from positive to negative during the night and back to positive the next day.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

previous grades' work with numbers — generalizing arithmetic in the process.

6.EE.B.7 When students write equations of the form $x + p = q$ and $px = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems.[62]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- Reading and transforming expressions involves seeing and making use of structure (MP.7). Relating expressions to situations requires making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2).
- The sequence of steps in the solution of an equation is a logical argument that students can construct and critique (MP.3). Such arguments require looking for and making use of structure (MP.7) and, over time, expressing regularity in repeated reasoning (MP.8).
- Thinking about the point (1, *r*) in a graph of a proportional relationship with unit rate *r* involves reasoning abstractly and quantitatively (MP.2). The graph models with mathematics (MP.4) and uses appropriate tools strategically (MP.5).
- Area, surface area, and volume present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).
- Students think with precision (MP.6) and reason quantitatively (MP.2) when they use variables to represent numbers and write expressions and equations to solve a problem (**6.EE.B.6–7**).

[62] For example, suppose Daniel went to visit his grandmother, who gave him \$5.50. Then he bought a book costing \$9.20 and had \$2.30 left. To find how much money he had before visiting his grandmother, an algebraic approach leads to the equation $x + 5.50 - 9.20 = 2.30$. An arithmetic approach without using variables at all would be to begin with 2.30, then add 9.20, then subtract 5.50. This yields the desired answer, but students will eventually encounter problems in which arithmetic approaches are unrealistically difficult and algebraic approaches must be used.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

- Working with data gives students an opportunity to use appropriate tools strategically (MP.5). For example, spreadsheets can be powerful for working with a data set with dozens or hundreds of data points.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning

■ B. Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

■ D. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

○ E. Compute fluently with multi-digit numbers and find common factors and multiples.

■ F. Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematics | Grade 6

PARCC Model Content Frameworks Indications (continued)

Expressions and Equations

- D. Apply and extend previous understandings of arithmetic to algebraic expressions.
- E. Reason about and solve one-variable equations and inequalities.
- F. Represent and analyze quantitative relationships between dependent and independent variables.

Ratios and Proportional Reasoning

- B. Understand ratio concepts and use ratio reasoning to solve problems.

Geometry

- B. Solve real-world and mathematical problems involving area, surface area and volume.

Statistics and Probability

- C. Develop understanding of statistical variability.
- D. Summarize and describe distributions.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Solve real-world and mathematical problems involving area, surface area, and volume: In this cluster, students work on problems with areas of triangles and volumes of right rectangular prisms, which connects to work in the Expressions and Equations domain. In addition, another standard within this cluster asks students to draw polygons in the coordinate plane, which supports other work with the coordinate plane in The Number System domain.

Grade 6

Ratios and Proportional Relationships (RP)

Understand ratio concepts and use ratio reasoning to solve problems	Major
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.2	Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System (NS)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Major
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
Compute fluently with multi-digit numbers and find common factors and multiples	**Additional**
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Grade 6

6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Apply and extend previous understandings of numbers to the system of rational numbers	**Major**
6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C >-7°C to express the fact that –3 °C is warmer than -7°C*. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* |–30| *= 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 6

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions	Major
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.
6.EE.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities	Major
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
6.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Grade 6

Represent and analyze quantitative relationships between dependent and independent variables	**Major**
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry (G)

Solve real-world and mathematical problems involving area, surface area, and volume	**Supporting**
6.G.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability (SP)

Develop understanding of statistical variability	**Additional**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Grade 6

Summarize and describe distributions		Additional
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	
6.SP.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

Grade 6

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Grade 7

In Grade 7, instruction should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 6 to Grade 7

- In grade 6, students learned about negative numbers and the kinds of quantities they can be used to represent; they also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will add, subtract, multiply, and divide within the system of rational numbers.
- Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.A.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.A.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (e.g., if a person walks $^1/_2$ mile in each $^1/_4$ hour, the unit rate is the complex fraction ½ / ¼ miles per hour or 2 miles per hour) (7.RP.A.1); and they analyze proportional relationships in geometric figures (7.G.A.1).
- Students solve a variety of problems involving angle measure, area, surface area, and volume (7.G.B.4–6).

Fluency Expectations or Examples of Culminating Standards

7.EE.B.3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.

7.EE.B.4 In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.A.1–3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.A.1).

7.NS.A.1-2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

Examples of Major Within-Grade Dependencies

- Meeting standard 7.EE.B.3 in its entirety will involve using rational number arithmetic (7.NS.A.1–3) and percents (7.RP.A.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.
- Because rational number arithmetic (7.NS.A.1–3) underlies the problem solving detailed in 7.EE.B.3 as well as the solution of linear expressions and equations (7.EE.A.1–2, 4), this work should likely begin at or near the start of the year.
- The work leading to meeting standards 7.EE.A.1–4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (7.NS.A.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (7.NS.A.2).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).
- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

Examples of Opportunities for In-Depth Focus

7.RP.A.2 Students in grade 7 grow in their ability to recognize, represent, and analyze proportional relationships in various ways, including by using tables, graphs, and equations.

7.NS.A.3 When students work toward meeting this standard (which is closely connected to 7.NS.A.1 and 7.NS.A.2), they consolidate their skill and understanding of addition, subtraction, multiplication and division of rational numbers.

7.EE.B.3 This is a major capstone standard for arithmetic and its applications.

7.EE.B.4 Work toward meeting this standard builds on the work that led to meeting 6.EE.B.7 and prepares students for the work that will lead to meeting 8.EE.C.7.

7.G.B.6 Work toward meeting this standard draws together grades 3–6 work with geometric measurement.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students compare arithmetic and algebraic solutions to the same problem (7.EE.B.4a), they are identifying correspondences between different approaches (MP.1).
- Solving an equation such as $4 = 8(x - 1/2)$ requires students to see and make use of structure (MP.7), temporarily viewing $x - 1/2$ as a single entity.
- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.A.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).
- Proportional relationships present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.

Content Emphases by Cluster[63]

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

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[63] Refer to the original PARCC Model Content Frameworks, located at www.parcconline.org/parcc-model-content-frameworks for additional support if needed.

Mathematics | Grade 7

PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning

■ B. Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

■ B. Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations

■ C. Use properties of operations to generate equivalent expressions.

■ D. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

○ C. Draw, construct and describe geometrical figures and describe the relationships between them.

○ D. Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability

◘ D. Use random sampling to draw inferences about a population.

○ E. Draw informal comparative inferences about two populations.

◘ F. Investigate chance processes and develop, use, and evaluate probability models.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.

- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.

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Ratios and Proportional Relationships (RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems		**Major**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*	
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.	
7.RP.3	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

The Number System (NS)

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers		**Major**
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	

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7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Expressions and Equations (EE)

Use properties of operations to generate equivalent expressions	**Major**

7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations	**Major**

7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

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Geometry (G)	
Draw, construct, and describe geometrical figures and describe the relationships between them	**Additional**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	**Additional**
7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
7.G.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability (SP)	
Use random sampling to draw inferences about a population	**Supporting**
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
Draw informal comparative inferences about two populations	**Additional**
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

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7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations*. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	
Investigate chance processes and develop, use, and, evaluate probability models		**Supporting**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

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Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

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In Grade 8, instruction should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square

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in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Grade 8

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a

Mathematics | Grade 8

PARCC Model Content Frameworks Indications (continued)

nonvertical line in the coordinate plane has equation $y = mx + b$.[64] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[65]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to

[64] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf.

[65] Note that the Geometry cluster "*Understand congruence and similarity using physical models, transparencies or geometry software*" supports high school work with congruent triangles and congruent figures.

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PARCC Model Content Frameworks Indications (continued)

grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $(y-2)/(x-1) = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).
- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).
- Solving an equation such as $3(x - {^1/_2}) = x + 2$ requires students to see and make use of structure (MP.7).
- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the

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PARCC Model Content Frameworks Indications (continued)

time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

The Number System	
◘	B. Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations	
■	D. Work with radicals and integer exponents.
■	E. Understand the connections between proportional relationships, lines and linear equations.
■	F. Analyze and solve linear equations and pairs of simultaneous linear equations.
Functions	
■	C. Define, evaluate and compare functions.
■	D. Use functions to model relationships between quantities.
Geometry	
■	D. Understand congruence and similarity using physical models, transparencies or geometry software.
■	E. Understand and apply the Pythagorean Theorem.
○	F. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability	
◘	B. Investigate patterns of association in bivariate data.

Mathematics | Grade 8

PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.
- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Grade 8

The Number System (NS)		
Know that there are numbers that are not rational, and approximate them by rational numbers		**Supporting**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	
Expressions and Equations (EE)		
Work with radicals and integer exponents		**Major**
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Understand the connections between proportional relationships, lines, and linear equations		**Major**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	

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Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
8.EE.8	Analyze and solve pairs of simultaneous linear equations. d. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. e. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* f. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Functions (F)

Define, evaluate, and compare functions		**Major**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]	
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	

Use functions to model relationships between quantities		**Major**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	

Grade 8

8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
Geometry (G)		
Understand congruence and similarity using physical models, transparencies, or geometry software		**Major**
8.G.1	Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	
Understand and apply the Pythagorean Theorem		**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres		**Additional**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	
Statistics and Probability (SP)		
Investigate patterns of association in bivariate data		**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	

Grade 8

8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

[1] Function notation is not required in Grade 8.

Grade 8

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Acceleration in Middle School

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school. Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

To prepare students for high school mathematics in eighth grade, based on guidance provided in A*ppendix A: Designing High School Mathematics Courses Based on the Common Core State Standards for Mathematics* ("Appendix A") the MDE has developed a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. These compacted courses are designed for districts offering the Traditional Pathway (Algebra I – Geometry – Algebra II) high school sequence, and the other for districts using an Integrated Pathway sequence (Integrated Math I – Integrated Math II – Integrated Math III) which is commonly found internationally. A snapshot of the content standards in each Pathway is available on pages 135-136. Both Pathways are based on the idea that content should compact 3 years of content into 2 years, at most. As a result, Grades 7, 8, and 9 were compacted into Grades 7 and 8 (a 3:2 compaction). Whereas, some 8th grade content is addressed in the 7th grade courses, and high school content is addressed in 8th grade.

The Compacted Traditional sequence compacts CCSSM Grade 7, CCSSM Grade 8, and high school CCSSM Algebra I into two years. Upon successfully completion of this Pathway, students will be ready for CCSSM Geometry or CCSSM Algebra II in high school. The Compacted Integrated sequencecompacts CCSSM Grade 7, CCSSM Grade 8, and CCSSM Integrated Mathematics I into two years. At the end of 8th grade, these students will be ready for CCSSM Integrated Mathematics II in high school. While the CCSS Grades K-7 effectively prepare students for algebra I in 8th grade, some standards from 8th grade have been placed in the Compacted Mathematics Grade 7 course to make the Compacted Mathematics Grade 8 courses more manageable-regardless of the Pathway chosen.

Appendix A presents a set of guidelines for the development of these compacted courses:

Acceleration in Middle School (continued)

1. Compacted courses should include the same Common Core State Standards as the non-compacted courses.

It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed.

Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning.

Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.

Traditionally, students taking high school mathematics in the eighth grade are expected to take a Pre-Calculus or Algebra III course in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career. (see Fourth Courses section of this paper for further detail).

SECONDARY SEQUENCE OPTIONS

Students will progress according to grade level through the sixth grade. Beginning in the seventh grade, students are given course sequence options based on academic progress, teacher recommendation, and parental consent. Below are suggested secondary course sequence options:

Suggested Secondary Course Sequence Options for Mathematics

Grade Level	OPTION 1	OPTION 2	OPTION 3
7	Grade 7	Compacted Grade 7	Compacted Grade 7
8	Grade 8	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)
9	Algebra I **or** Integrated Math I	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math II
10	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math III	Geometry **or** Integrated Math III
11	Algebra II **or** Integrated Math III	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course
12	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course

PARCC Traditional Pathway Summary Table:

ALGEBRA I—GEOMETRY—ALGEBRA II

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. Algebra I and II are adjacent so as to make the shading continuous, despite the fact that some states offer these courses a year apart.

	CCSSM Standard	A I	A II	G
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3	▪		
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1	▪		
	A-SSE.A.2	▪	▪	
	A-SSE.B.3a	▪		
	A-SSE.B.3b	▪		
	A-SSE.B.3c	▪	▪	
	A-SSE.B.4		▪	
	A-APR.A.1	▪		
	A-APR.B.2		▪	
	A-APR.B.3	▪	▪	
	A-APR.C.4		▪	
	A-APR.D.6		▪	
	A-CED.A.1	▪	▪	
	A-CED.A.2	▪		
	A-CED.A.3	▪		
	A-CED.A.4	▪		
	A-REI.A.1	▪	▪	
	A-REI.A.2		▪	
	A-REI.B.3	▪		
	A-REI.B.4a	▪		
	A-REI.B.4b	▪	▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪	▪	
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪	▪	
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪	▪	
	F-IF.B.4	▪	▪	
	F-IF.B.5	▪		
	F-IF.B.6	▪	▪	
	F-IF.C.7a	▪		
	F-IF.C.7b	▪		
	F-IF.C.7c		▪	
	F-IF.C.7e		▪	
	F-IF.C.8a	▪		
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2		▪	
	F-BF.B.3	▪	▪	
	F-BF.B.4a		▪	
	F-LE.A.1	▪		
	F-LE.A.2	▪	▪	
	F-LE.A.3	▪		
	F-LE.A.4		▪	
	F-LE.B.5	▪	▪	
	F-TF.A.1		▪	
	F-TF.A.2		▪	
	F-TF.B.5		▪	
	F-TF.C.8		▪	

	CCSSM Standard	A I	A II	G
Geometry	G-CO.A.1			▪
	G-CO.A.2			▪
	G-CO.A.3			▪
	G-CO.A.4			▪
	G-CO.A.5			▪
	G-CO.B.6			▪
	G-CO.B.7			▪
	G-CO.B.8			▪
	G-CO.C.9			▪
	G-CO.C.10			▪
	G-CO.C.11			▪
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1			▪
	G-SRT.A.2			▪
	G-SRT.A.3			▪
	G-SRT.B.4			▪
	G-SRT.B.5			▪
	G-SRT.C.6			▪
	G-SRT.C.7			▪
	G-SRT.C.8			▪
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2		▪	
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1			▪
	G-GMD.A.3			▪
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4		▪	
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	
	S-ID.B.6b	▪		
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1		▪	
	S-IC.A.2		▪	
	S-IC.B.3		▪	
	S-IC.B.4		▪	
	S-IC.B.5		▪	
	S-IC.B.6		▪	
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

PARCC Integrated Pathway Summary Table:

INTEGRATED MATH I—INTEGRATED MATH II-INTEGRATED MATH III

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course.

	CCSSM Standard	M I	M II	M III
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3		▪	
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	▪
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1a	▪		
	A-SSE.A.1b	▪	▪	
	A-SSE.A.2		▪	▪
	A-SSE.B.3a		▪	
	A-SSE.B.3b		▪	
	A-SSE.B.3c	▪		
	A-SSE.B.4			▪
	A-APR.A.1		▪	
	A-APR.B.2			▪
	A-APR.B.3			▪
	A-APR.C.4			▪
	A-APR.D.6			▪
	A-CED.A.1	▪	▪	▪
	A-CED.A.2	▪	▪	▪
	A-CED.A.3	▪		
	A-CED.A.4	▪	▪	
	A-REI.A.1		▪	▪
	A-REI.A.2			▪
	A-REI.B.3	▪		
	A-REI.B.4a		▪	
	A-REI.B.4b		▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪		
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪		▪
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪		
	F-IF.B.4	▪	▪	▪
	F-IF.B.5	▪	▪	
	F-IF.B.6	▪	▪	▪
	F-IF.C.7a	▪	▪	
	F-IF.C.7b		▪	
	F-IF.C.7c			▪
	F-IF.C.7e		▪	▪
	F-IF.C.8a		▪	
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	▪
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2	▪		
	F-BF.B.3		▪	▪
	F-BF.B.4a			▪
	F-LE.A.1a	▪		
	F-LE.A.1b	▪		
	F-LE.A.1c	▪		
	F-LE.A.2	▪		
	F-LE.A.3	▪		
	F-LE.A.4			▪
	F-LE.B.5	▪		
	F-TF.A.1			▪
	F-TF.A.2			▪
	F-TF.B.5			▪
	F-TF.C.8			▪

	CCSSM Standard	M I	M II	M III
Geometry	G-CO.A.1	▪		
	G-CO.A.2	▪		
	G-CO.A.3	▪		
	G-CO.A.4	▪		
	G-CO.A.5	▪		
	G-CO.B.6	▪		
	G-CO.B.7	▪		
	G-CO.B.8	▪		
	G-CO.C.9	▪		
	G-CO.C.10	▪		
	G-CO.C.11	▪		
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1a		▪	
	G-SRT.A.1b		▪	
	G-SRT.A.2		▪	
	G-SRT.A.3		▪	
	G-SRT.B.4		▪	
	G-SRT.B.5		▪	
	G-SRT.C.6		▪	
	G-SRT.C.7		▪	
	G-SRT.C.8		▪	
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2			▪
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1		▪	
	G-GMD.A.3		▪	
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4			▪
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	▪
	S-ID.B.6b		▪	▪
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1			▪
	S-IC.A.2			▪
	S-IC.B.3			▪
	S-IC.B.4			▪
	S-IC.B.5			▪
	S-IC.B.6			▪
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

Mathematics | High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students might learn in order to take advanced courses are included in the Advanced Mathematics Plus and Algebra III courses.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School —Number and Quantity Conceptual Category

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Algebra Conceptual Category

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Mathematics | High School—Algebra Conceptual Category (continued)

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

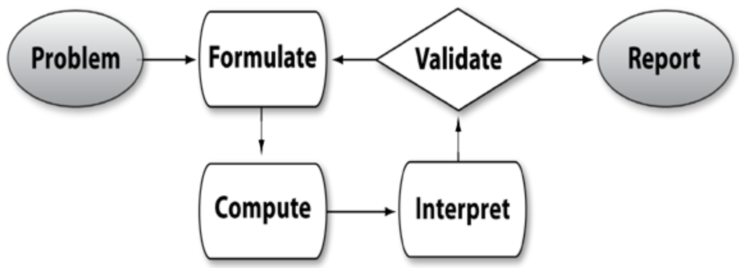
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and

Mathematics | High School—Modeling Conceptual Category (continued)

other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Mathematics | High School—Modeling Conceptual Category (continued)

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).*

Mathematics | High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many

Mathematics | High School—Geometry Conceptual Category (continued)

real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Statistics and Probability* Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematics | Compacted Mathematics Grade 7

In Compacted Mathematics Grade 7, a one-credit course, instruction should focus on four critical areas from Grade 7: (1) applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. Each critical area is described below.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematics | Compacted Mathematics Grade 7 (continued)

In Compacted Mathematics Grade 7, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles

Mathematics | Compacted Mathematics Grade 7 (continued)

because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 6 to Grade 7

- In grade 6, students learned about negative numbers and the kinds of quantities they can be used to represent; they also learned about absolute value and ordering of rational numbers, including in real-world contexts. In grade 7, students will add, subtract, multiply, and divide within the system of rational numbers.
- Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.A.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.A.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (e.g., if a person walks $1/2$ mile in each $1/4$ hour, the unit rate is the complex fraction $(\frac{1}{2}) / (\frac{1}{4})$ miles per hour or 2 miles per hour) (7.RP.A.1); and they analyze proportional relationships in geometric figures (7.G.A.1).
- Students solve a variety of problems involving angle measure, area, surface area, and volume (7.G.B.4–6).

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.
- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Fluency Expectations or Examples of Culminating Standards

7.EE.B.3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

7.EE.B.4 In solving word problems leading to one-variable equations of the form $px + q = r$ and $p(x + q) = r$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.A.1–3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.A.1).

7.NS.A.1-2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8, expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in grade 7.

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- Meeting standard 7.EE.B.3 in its entirety will involve using rational number arithmetic (7.NS.A.1–3) and percents (7.RP.A.3). Work leading to meeting this standard could be organized as a recurring activity that tracks the students' ongoing acquisition of new skills in rational number arithmetic and percents.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

- Because rational number arithmetic (7.NS.A.1–3) underlies the problem solving detailed in 7.EE.B.3 as well as the solution of linear expressions and equations (7.EE.A.1–2, 4), this work should likely begin at or near the start of the year.

- The work leading to meeting standards 7.EE.A.1–4 could be divided into two phases, one centered on addition and subtraction (e.g., solving $x + q = r$) in relation to rational number addition and subtraction (7.NS.A.1) and another centered on multiplication and division (e.g., solving $px + q = r$ and $p(x + q) = r$) in relation to rational number multiplication and division (7.NS.A.2).

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[66] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[67]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).

- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

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[66] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[67] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Examples of Opportunities for In-Depth Focus

7.RP.A.2 Students in grade 7 grow in their ability to recognize, represent, and analyze proportional relationships in various ways, including by using tables, graphs, and equations.

7.NS.A.3 When students work toward meeting this standard (which is closely connected to 7.NS.A.1 and 7.NS.A.2), they consolidate their skill and understanding of addition, subtraction, multiplication and division of rational numbers.

7.EE.B.3 This is a major capstone standard for arithmetic and its applications.

7.EE.B.4 Work toward meeting this standard builds on the work that led to meeting 6.EE.B.7 and prepares students for the work that will lead to meeting 8.EE.C.7.

7.G.B.6 Work toward meeting this standard draws together grades 3–6 work with geometric measurement.

When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students compare arithmetic and algebraic solutions to the same problem (7.EE.B.4a), they are identifying correspondences between different approaches (MP.1).
- Solving an equation such as $4 = 8(x - \frac{1}{2})$requires students to see and make use of structure (MP.7), temporarily viewing $x - \frac{1}{2}$ as a single entity.
- When students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.A.2), they have an opportunity to construct viable arguments and critique the reasoning of others (MP.3). Such problems also present opportunities for using appropriate tools strategically (MP.5).
- Proportional relationships present opportunities for modeling (MP.4). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes.

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y - 2)}/_{(x - 1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).
- The Pythagorean Theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

- Solving an equation such as $3(x - {}^{1}/_{2}) = x + 2$ requires students to see and make use of structure (MP.7).
- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

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PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 7

Ratios and Proportional Reasoning

■ B. Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

■ B. Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations

■ C. Use properties of operations to generate equivalent expressions.

■ D. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

○ C. Draw, construct and describe geometrical figures and describe the relationships between them.

○ D. Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability

◘ D. Use random sampling to draw inferences about a population.

○ E. Draw informal comparative inferences about two populations.

◘ F. Investigate chance processes and develop, use, and evaluate probability models.

Grade 8

The Number System

◘ B. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ D. Work with radicals and integer exponents.

■ E. Understand the connections between proportional relationships, lines and linear equations.

■ F. Analyze and solve linear equations and pairs of simultaneous linear equations.

Mathematics | Compacted Mathematics Grade 7

PARCC Model Content Frameworks Indications (continued)

Grade 8 (continued)

Functions

- C. Define, evaluate and compare functions.
- D. Use functions to model relationships between quantities.

Geometry

- D. Understand congruence and similarity using physical models, transparencies or geometry software.
- E. Understand and apply the Pythagorean Theorem.
- F. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- C. Investigate patterns of association in bivariate data.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.

- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Compacted Mathematics Grade 7

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems		**Major**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*	
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	
7.RP.3	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers		**Major**
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	

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7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p/q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.[1]
Know that there are numbers that are not rational, and approximate them by rational numbers	**Supporting**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations

Use properties of operations to generate equivalent expressions	**Major**
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	**Major**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

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7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. b. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	
Work with radicals and integer exponents		**Major**
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Understand the connections between proportional relationships, lines, and linear equations		**Major**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	

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Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
Geometry		
Draw, construct, and describe geometrical figures and describe the relationships between them		**Additional**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume		**Additional**
7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	
7.G.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	
Understand congruence and similarity using physical models, transparencies, or geometry software		**Major**
8.G.1	Verify experimentally the properties of rotations, reflections, and translations a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	

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8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres		**Additional**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	
Statistics and Probability		
Use random sampling to draw inferences about a population		**Supporting**
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	
Draw informal comparative inferences about two populations		**Additional**
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	
7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations*. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	

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Investigate chance processes and develop, use, and, evaluate probability models	**Supporting**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Compacted Mathematics Grade 7

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

In Compacted Mathematics Grade 8 (with Algebra I), a one-credit course, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles

Mathematics | Compacted Mathematics Grade 8 (with Algebra I) (continued)

because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

In Algebra I, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. Instruction should focus on five critical areas: (1) analyze and explain the process of solving equations and inequalities: (2) learn function notation and develop the concepts of domain and range; (3) use regression techniques; (4) create quadratic and exponential expressions; and (5) select from among these functions to model phenomena. Each critical area is described below.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I) (continued)

(3) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(4) In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

(5) In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability.** Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4-6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (N-Q).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (6.EE.A.3, 7.EE.A.1). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (7.EE.A.2). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[68]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions.

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[68] See, for example, "Mindful Manipulation," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

- Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.
- Students in grade 8 connected their knowledge about proportional relationships, lines, and linear equations (8.EE.B.5, 6). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:
    - The graph of any linear equation in two variables is a line.
    - Any line is the graph of a linear equation in two variables.
- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.B.6). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.
- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K-8. This expands problem solving from grades K-8 dramatically.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[69] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[70]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

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[69] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[70] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

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PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y-2)}/_{(x-1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).

- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

- Solving an equation such as $3(x - ½) = x + 2$ requires students to see and make use of structure (MP.7).

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PARCC Model Content Frameworks Indications (continued)

- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.

- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them** (MP.1).

- **Model with mathematics** (MP.4).

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively** (MP.2). This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (A-CED, F-BF) and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.

- **Use appropriate tools strategically** (MP.5). Spreadsheets, a function modeling language, graphing tools, and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.

- **Attend to precision** (MP.6). In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (A-CED, A-REI) helps students understand the idea in new ways.

- **Look for and make use of structure** (MP.7). For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$, a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3)((7x) + 2)$ (A-SSE, A-APR).

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PARCC Model Content Frameworks Indications (continued)

- **Look for and express regularity in repeated reasoning** (MP.8). Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (A-CED). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans.

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

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PARCC Model Content Frameworks Indications (continued)

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 8

The Number System

◘ B. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ D. Work with radicals and integer exponents.

■ E. Understand the connections between proportional relationships, lines and linear equations.

■ F. Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

■ C. Define, evaluate and compare functions.

■ D. Use functions to model relationships between quantities.

Geometry

■ D. Understand congruence and similarity using physical models, transparencies or geometry software.

■ E. Understand and apply the Pythagorean Theorem.

○ F. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

◘ B. Investigate patterns of association in bivariate data.

Algebra I

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

○ D. Use properties of rational and irrational numbers (3)

Quantities* (N-Q)

◘ B. Reason quantitatively and use units to solve problems (1, <u>2</u>, 3)

Seeing Structure in Expressions (A-SSE)

■ C. Interpret the structure of expressions (1, <u>2</u>)

◘ D. Write expressions in equivalent forms to solve problems (<u>3</u>)

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PARCC Model Content Frameworks Indications (continued)

Algebra I (continued)

Arithmetic with Polynomials and Rational Expressions (A-APR)

- C. Perform arithmetic operations on polynomials (1)
- D. Understand the relationship between zeros and factors of polynomials (3)

Creating Equations* (A-CED)

- B. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

- E. Understand solving equations as a process of reasoning and explain the reasoning (1)
- F. Solve equations and inequalities in one variable (3, 4)
- G. Solve systems of equations (5, 6)
- H. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

- D. Understand the concept of a function and use function notation (1, 2, 3)
- E. Interpret functions that arise in applications in terms of the context (4, 5, 6)
- F. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

- C. Build a function that models a relationship between two quantities (1)
- D. Build new functions from existing functions (3)

Linear, Quadratic, and Exponential Models* (F-LE)

- C. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- D. Interpret expressions for functions in terms of the situation they model (5)

Interpreting Categorical and Quantitative Data (S-ID)

- D. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- E. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- F. Interpret linear models (7, 8, 9)

Mathematics | Compacted Mathematics Grade 8 (with Algebra I)

PARCC Model Content Frameworks Indications (continued)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (**8.NS.A.1–2**) is intimately related to work with radicals (**8.EE.A.2**), and both of these may be connected to the Pythagorean theorem (**8.G**, second cluster) as well as to volume problems (**8.G.C.9**), e.g., in which a cube has known volume but unknown edge lengths.

- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Fluency Recommendations for Algebra I

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Compacted Mathematics Grade 8 (with Algebra I)

Number and Quantity

The Real Number System (N-RN)

Use properties of rational and irrational numbers		**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	

Quantities (N-Q)*

Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	

Algebra

Expressions and Expressions (EE)

Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.8	Analyze and solve pairs of simultaneous linear equations. d. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. e. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.* f. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* c. Interpret parts of an expression, such as terms, factors, and coefficients. d. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$.	

Compacted Mathematics Grade 8 (with Algebra I)

Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* d. Factor a quadratic expression to reveal the zeros of the function it defines. e. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. f. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Perform arithmetic operations on polynomials		**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Understand the relationship between zeros and factors of polynomials		**Supporting**
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	

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Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
A-REI.4	Solve quadratic equations in one variable. e. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. f. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Functions		
Functions (F)		
Define, evaluate, and compare functions		**Major**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]	
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	

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8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	
	Use functions to model relationships between quantities	**Major**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	
	Interpreting Functions (F-IF)	
	Understand the concept of a function and use function notation	**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	
	Interpret functions that arise in applications in terms of the context	**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	

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Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph linear and quadratic functions and show intercepts, maxima, and minima. d. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* c. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* d. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. e. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. f. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	

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F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
Interpret expressions for functions in terms of the situation they model		**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
Geometry		
Geometry (G)		
Understand and apply the Pythagorean Theorem		**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
Statistics and Probability		
Statistics and Probability (SP)		
Investigate patterns of association in bivariate data		**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	

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S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* d. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* e. Informally assess the fit of a function by plotting and analyzing residuals. f. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

[1] Function notation is not required at Grade 8

* Modeling Standards (**High School standards only**)

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Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | Compacted Mathematics Grade 8 (with Integrated Math I)

In Compacted Mathematics Grade 8 (with Integrated Math I), a one-credit course, instruction should focus on three critical areas from Grade 8: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Each critical area is described below.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a

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straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

In Integrated Mathematics I, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. The critical areas deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Integrated Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Each critical area is described below.

(1) By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are

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measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

(4) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(5) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(6) Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

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The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

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PARCC Model Content Frameworks Indications

Examples of Key Advances from Grade 7 to Grade 8

- Students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a nonvertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line as well as the unit rate of a proportional relationship (in the case $b = 0$). Students also formalize their previous work with linear relationships by working with functions — rules that assign to each input exactly one output.

- By working with equations such as $x^2 = 2$ and in geometric contexts such as the Pythagorean theorem, students enlarge their concept of number beyond the system of rationals to include irrational numbers. They represent these numbers with radical expressions and approximate these numbers with rationals.

Examples of Key Advances from Grades K–8

- Students build on previous work with solving linear equations and systems of linear equations in two ways: (a) They extend to more formal solution methods, including attending to the structure of linear expressions, and (b) they solve linear inequalities.

- Students formalize their understanding of the definition of a function, particularly their understanding of linear functions, emphasizing the structure of linear expressions. Students also begin to work on exponential functions, comparing them to linear functions.

- Work with congruence and similarity motions that started in grades 6-8 progresses. Students also consider sufficient conditions for congruence of triangles.

- Work with the bivariate data and scatter plots in grades 6-8 is extended to working with lines of best fit.

Fluency Expectations or Examples of Culminating Standards

8.EE.C.7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.

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PARCC Model Content Frameworks Indications (continued)

8.G.C.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres — together with their previous grade 7 work in angle measure, area, surface area and volume (7.G.B.4–6) — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.B.3), can be combined and used in flexible ways as part of modeling during high school — not to mention after high school for college and careers.

Examples of Major Within-Grade Dependencies

- An important development takes place in grade 8 when students make connections between proportional relationships, lines, and linear equations (8.EE, **second cluster**). Making these connections depends on prior grades' work, including 7.RP.A.2 and 6.EE.C.9. There is also a major dependency within grade 8 itself: The angle-angle criterion for triangle similarity underlies the fact that a nonvertical line in the coordinate plane has equation $y = mx + b$.[71] Therefore, students must do work with congruence and similarity (8.G.A.1–5) before they are able to justify the connections among proportional relationships, lines, and linear equations. Hence the indicated geometry work should likely begin at or near the very start of the year.[72]

- Much of the work of grade 8 involves lines, linear equations, and linear functions (8.EE.B.5–8; 8.F.A.3–4; 8.SP.A.2–3). Irrational numbers, radicals, the Pythagorean theorem, and volume (8.NS.A.1–2; 8.EE.A.2; 8.G.B.6–9) are nonlinear in nature. Curriculum developers might choose to address linear and nonlinear bodies of content somewhat separately. An exception, however, might be that when addressing functions, pervasively treating linear functions as separate from nonlinear functions might obscure the concept of function *per se*. There should also be sufficient treatment of nonlinear functions to avoid giving students the misleading impression that all functional relationships are linear (see also 7.RP.A.2a).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

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[71] See page 12 of the *Progression for Expressions and Equations:* http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

[72] Note that the Geometry cluster "Understand congruence and similarity using physical models, transparencies or geometry software" supports high school work with congruent triangles and congruent figures.

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PARCC Model Content Frameworks Indications (continued)

Examples of Opportunities for In-Depth Focus

8.EE.B.5 When students work toward meeting this standard, they build on grades 6–7 work with proportions and position themselves for grade 8 work with functions and the equation of a line.

8.EE.C.7 This is a culminating standard for solving one-variable linear equations.

8.EE.C.8 When students work toward meeting this standard, they build on what they know about two-variable linear equations, and they enlarge the varieties of real-world and mathematical problems they can solve.

8.F.A.2 Work toward meeting this standard repositions previous work with tables and graphs in the new context of input/output rules.

8.G.B.7 The Pythagorean theorem is useful in practical problems, relates to grade-level work in irrational numbers and plays an important role mathematically in coordinate geometry in high school.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded, and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow.

- When students convert a fraction such as $^1/_7$ to a decimal, they might notice that they are repeating the same calculations and conclude that the decimal repeats. Similarly, by repeatedly checking whether points are on a line through (1, 2) with slope 3, students might abstract the equation of the line in the form $^{(y-2)}/_{(x-1)} = 3$. In both examples, students look for and express regularity in repeated reasoning (MP.8).

- The Pythagorean theorem can provide opportunities for students to construct viable arguments and critique the reasoning of others (e.g., if a student in the class seems to be confusing the theorem with its converse) (MP.3).

- Solving an equation such as $3(x - {}^1/_2) = x + 2$ requires students to see and make use of structure (MP.7).

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PARCC Model Content Frameworks Indications (continued)\

- Much of the mathematics in grade 8 lends itself to modeling (MP.4). For example, standard 8.F.B.4 involves modeling linear relationships with functions.
- Scientific notation (8.EE.A.4) presents opportunities for strategically using appropriate tools (MP.5). For example, the computation $(1.73 \times 10^{-4}) \cdot (1.73 \times 10^{-5})$ can be done quickly with a calculator by squaring 1.73 and then using properties of exponents to determine the exponent of the product by inspection.
    - **Modeling with mathematics** (MP.4) should be a particular focus as students see the purpose and meaning for working with linear and exponential equations and functions.
    - **Using appropriate tools strategically** (MP.5) is also important as students explore those models in a variety of ways, including with technology. For example, students might be given a set of data points and experiment with graphing a line that fits the data.
    - As Mathematics I continues to develop a foundation for more formal reasoning, students should engage in the practice of **constructing viable arguments and critiquing the reasoning of others** (MP.3).

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. However, the assessments will strongly focus where the standards strongly focus.

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PARCC Model Content Frameworks Indications (continued)

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given following the table on the next page for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it, in ways that foster greater focus and coherence.

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Grade 8

The Number System

◘ B. Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

■ D. Work with radicals and integer exponents.

■ E. Understand the connections between proportional relationships, lines and linear equations.

■ F. Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

■ C. Define, evaluate and compare functions.

■ D. Use functions to model relationships between quantities.

Geometry

■ D. Understand congruence and similarity using physical models, transparencies or geometry software.

■ E. Understand and apply the Pythagorean Theorem.

○ F. Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

◘ E. Investigate patterns of association in bivariate data.

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PARCC Model Content Frameworks Indications (continued)

Integrated Math I
Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◻ Supporting Content; ○ Additional Content.

Quantities* (N-Q)

◻ B. Reason quantitatively and use units to solve problems (1, <u>2</u>, 3)

Seeing Structure in Expressions (A-SSE)

■ C. Interpret the structure of expressions (<u>1</u>)

◻ D. Write expressions in equivalent forms to solve problems (<u>3</u>)

Creating Equations* (A-CED)

■ B. Create equations that describe numbers or relationships (<u>1</u>, <u>2</u>, 3, <u>4</u>)

Reasoning with Equations and Inequalities (A-REI)

■ F. Solve equations and inequalities in one variable (3)

○ G. Solve systems of equations (5, 6)

■ H. Represent and solve equations and inequalities graphically (10, <u>11</u>, 12)

Interpreting Functions (F-IF)

■ D. Understand the concept of a function and use function notation (1, 2, 3)

■ E. Interpret functions that arise in applications in terms of the context (<u>4</u>, <u>5</u>, <u>6</u>)

◻ F. Analyze functions using different representations (<u>7</u>, <u>9</u>)

Building Functions (F-BF)

■ C. Build a function that models a relationship between two quantities (<u>1</u>, 2)

Linear, Quadratic, and Exponential Models* (F-LE)

◻ C. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)

◻ D. Interpret expressions for functions in terms of the situation they model (5)

Congruence (G-CO)

◻ D. Experiment with transformations in the plane (1, 2, 3, 4, 5)

■ E. Understand congruence in terms of rigid motions (6, 7, 8)

■ F. Prove geometric theorems (9, 10, 11)

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PARCC Model Content Frameworks Indications

Integrated Math I (continued)

Interpreting Categorical and Quantitative Data (S-ID)

- D. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- E. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- F. Interpret linear models (7, 8, 9)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Know that there are numbers that are not rational, and approximate them by rational numbers: Work with the number system in this grade (8.NS.A.1–2) is intimately related to work with radicals (8.EE.A.2), and both of these may be connected to the Pythagorean theorem (8.G, **second cluster**) as well as to volume problems (8.G.C.9), e.g., in which a cube has known volume but unknown edge lengths.
- Investigate patterns of association in bivariate data: Looking for patterns in scatterplots and using linear models to describe data are directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice, MP.4: Model with mathematics.

Fluency Recommendations for Integrated Math I

A/G High school students should become fluent in solving characteristic problems involving the analytic geometry of lines, such as finding the equation of a line given a point and a slope. This fluency can support students in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

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Number and Quantity		
Quantities (N-Q)*		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	
Algebra		
Expressions and Expressions (EE)		
Analyze and solve linear equations and pairs of simultaneous linear equations		**Major**
8.EE.8	Analyze and solve pairs of simultaneous linear equations. d. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. e. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* f. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* d. Interpret parts of an expression, such as terms, factors, and coefficients. e. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* f. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

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Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

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Functions	
Functions (F)	
Define, evaluate, and compare functions	**Major**

8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [1]
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities	**Major**

8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Interpreting Functions (F-IF)	
Understand the concept of a function and use function notation	**Major**

F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

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Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* b. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* d. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. e. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. f. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	

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F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
	Interpret expressions for functions in terms of the situation they model	**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
	Geometry	
	Geometry (G)	
	Understand and apply the Pythagorean Theorem	**Major**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
	Congruence (G-CO)	
	Experiment with transformations in the plane	**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
	Understand congruence in terms of rigid motions	**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	

Compacted Mathematics Grade 8 (with Integrated Math I)

G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
	Prove geometric theorems	**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	
	Statistics and Probability	
	Statistics and Probability (SP)	
	Investigate patterns of association in bivariate data	**Supporting**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	

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Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Supporting**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Additional**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* b. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* C. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

[1] Function notation is not required at Grade 8

* Modeling Standards (**High School standards only**)

Compacted Mathematics Grade 8 (with Integrated Math I)

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

College- and Career- Readiness Standards for Mathematics (Grades 9-12)

SECONDARY SEQUENCE OPTIONS

Students will progress according to grade level through the sixth grade. Beginning in the seventh grade, students are given course sequence options based on academic progress, teacher recommendation, and parental consent. Below are suggested secondary course sequence options:

Suggested Secondary Course Sequence Options for Mathematics

Grade Level	OPTION 1	OPTION 2	OPTION 3
7	Grade 7	Compacted Grade 7	Compacted Grade 7
8	Grade 8	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)	Compacted Grade 8 (with Algebra I) **or** Compacted Grade 8 (with Integrated Math I)
9	Algebra I **or** Integrated Math I	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math II
10	Geometry **or** Integrated Math II	Algebra II **or** Integrated Math III	Geometry **or** Integrated Math III
11	Algebra II **or** Integrated Math III	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course
12	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course	Algebra III, Advanced Mathematics Plus, Calculus, AP Calculus, AP Statistics, Dual Credit/Dual Enrollment or SREB Math Ready Course

PARCC TRADITIONAL PATHWAY SUMMARY TABLE:
ALGEBRA I—GEOMETRY—ALGEBRA II

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. <u>Algebra I and II are adjacent so as to make the shading continuous, despite the fact that some states offer these courses a year apart.</u>

	CCSSM Standard	A I	A II	G
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3	▪		
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1	▪		
	A-SSE.A.2	▪	▪	
	A-SSE.B.3a	▪		
	A-SSE.B.3b	▪		
	A-SSE.B.3c	▪	▪	
	A-SSE.B.4		▪	
	A-APR.A.1	▪		
	A-APR.B.2		▪	
	A-APR.B.3	▪	▪	
	A-APR.C.4		▪	
	A-APR.D.6		▪	
	A-CED.A.1	▪	▪	
	A-CED.A.2	▪		
	A-CED.A.3	▪		
	A-CED.A.4	▪		
	A-REI.A.1	▪	▪	
	A-REI.A.2		▪	
	A-REI.B.3	▪		
	A-REI.B.4a	▪		
	A-REI.B.4b	▪	▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪	▪	
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪	▪	
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪	▪	
	F-IF.B.4	▪	▪	
	F-IF.B.5	▪		
	F-IF.B.6	▪	▪	
	F-IF.C.7a	▪		
	F-IF.C.7b	▪		
	F-IF.C.7c		▪	
	F-IF.C.7e		▪	
	F-IF.C.8a	▪		
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2		▪	
	F-BF.B.3	▪	▪	
	F-BF.B.4a		▪	
	F-LE.A.1	▪		
	F-LE.A.2	▪	▪	
	F-LE.A.3	▪		
	F-LE.A.4		▪	
	F-LE.B.5	▪	▪	
	F-TF.A.1		▪	
	F-TF.A.2		▪	
	F-TF.B.5		▪	
	F-TF.C.8		▪	

	CCSSM Standard	A I	A II	G
Geometry	G-CO.A.1			▪
	G-CO.A.2			▪
	G-CO.A.3			▪
	G-CO.A.4			▪
	G-CO.A.5			▪
	G-CO.B.6			▪
	G-CO.B.7			▪
	G-CO.B.8			▪
	G-CO.C.9			▪
	G-CO.C.10			▪
	G-CO.C.11			▪
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1			▪
	G-SRT.A.2			▪
	G-SRT.A.3			▪
	G-SRT.B.4			▪
	G-SRT.B.5			▪
	G-SRT.C.6			▪
	G-SRT.C.7			▪
	G-SRT.C.8			▪
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2		▪	
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1			▪
	G-GMD.A.3			▪
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4		▪	
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	
	S-ID.B.6b	▪		
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1		▪	
	S-IC.A.2		▪	
	S-IC.B.3		▪	
	S-IC.B.4		▪	
	S-IC.B.5		▪	
	S-IC.B.6		▪	
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

PARCC Integrated Pathway Summary Table:
Integrated Math I—Integrated Math II-Integrated Math III

This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course.

	CCSSM Standard	M I	M II	M III
Number & Quantity	N-RN.A.1		▪	
	N-RN.A.2		▪	
	N-RN.B.3		▪	
	N-Q.A.1	▪		
	N-Q.A.2	▪	▪	▪
	N-Q.A.3	▪		
	N-CN.A.1		▪	
	N-CN.A.2		▪	
	N-CN.C.7		▪	
Algebra	A-SSE.A.1a	▪		
	A-SSE.A.1b	▪	▪	
	A-SSE.A.2		▪	▪
	A-SSE.B.3a		▪	
	A-SSE.B.3b		▪	
	A-SSE.B.3c	▪		
	A-SSE.B.4			▪
	A-APR.A.1		▪	
	A-APR.B.2			▪
	A-APR.B.3			▪
	A-APR.C.4			▪
	A-APR.D.6			▪
	A-CED.A.1	▪	▪	▪
	A-CED.A.2	▪	▪	▪
	A-CED.A.3	▪		
	A-CED.A.4	▪	▪	
	A-REI.A.1		▪	▪
	A-REI.A.2			▪
	A-REI.B.3	▪		
	A-REI.B.4a		▪	
	A-REI.B.4b		▪	
	A-REI.C.5	▪		
	A-REI.C.6	▪		
	A-REI.C.7		▪	
	A-REI.D.10	▪		
	A-REI.D.11	▪		▪
	A-REI.D.12	▪		
Functions	F-IF.A.1	▪		
	F-IF.A.2	▪		
	F-IF.A.3	▪		
	F-IF.B.4	▪	▪	▪
	F-IF.B.5	▪	▪	
	F-IF.B.6	▪	▪	▪
	F-IF.C.7a	▪	▪	
	F-IF.C.7b		▪	
	F-IF.C.7c			▪
	F-IF.C.7e		▪	▪
	F-IF.C.8a		▪	
	F-IF.C.8b		▪	
	F-IF.C.9	▪	▪	▪
	F-BF.A.1a	▪	▪	
	F-BF.A.1b		▪	
	F-BF.A.2	▪		
	F-BF.B.3		▪	▪
	F-BF.B.4a			▪
	F-LE.A.1a	▪		
	F-LE.A.1b	▪		
	F-LE.A.1c	▪		
	F-LE.A.2	▪		
	F-LE.A.3	▪		
	F-LE.A.4			▪
	F-LE.B.5	▪		
	F-TF.A.1			▪
	F-TF.A.2			▪
	F-TF.B.5			▪
	F-TF.C.8			▪

	CCSSM Standard	M I	M II	M III
Geometry	G-CO.A.1	▪		
	G-CO.A.2	▪		
	G-CO.A.3	▪		
	G-CO.A.4	▪		
	G-CO.A.5	▪		
	G-CO.B.6	▪		
	G-CO.B.7	▪		
	G-CO.B.8	▪		
	G-CO.C.9	▪		
	G-CO.C.10	▪		
	G-CO.C.11	▪		
	G-CO.D.12			▪
	G-CO.D.13			▪
	G-SRT.A.1a		▪	
	G-SRT.A.1b		▪	
	G-SRT.A.2		▪	
	G-SRT.A.3		▪	
	G-SRT.B.4		▪	
	G-SRT.B.5		▪	
	G-SRT.C.6		▪	
	G-SRT.C.7		▪	
	G-SRT.C.8		▪	
	G-C.A.1			▪
	G-C.A.2			▪
	G-C.A.3			▪
	G-C.B.5			▪
	G-GPE.A.1			▪
	G-GPE.A.2			▪
	G-GPE.B.4			▪
	G-GPE.B.5			▪
	G-GPE.B.6			▪
	G-GPE.B.7			▪
	G-GMD.A.1		▪	
	G-GMD.A.3		▪	
	G-GMD.B.4			▪
	G-MG.A.1			▪
	G-MG.A.2			▪
	G-MG.A.3			▪
Statistics & Probability	S-ID.A.1	▪		
	S-ID.A.2	▪		
	S-ID.A.3	▪		
	S-ID.A.4			▪
	S-ID.B.5	▪		
	S-ID.B.6a	▪	▪	▪
	S-ID.B.6b		▪	▪
	S-ID.B.6c	▪		
	S-ID.C.7	▪		
	S-ID.C.8	▪		
	S-ID.C.9	▪		
	S-IC.A.1			▪
	S-IC.A.2			▪
	S-IC.B.3			▪
	S-IC.B.4			▪
	S-IC.B.5			▪
	S-IC.B.6			▪
	S-CP.A.1		▪	
	S-CP.A.2		▪	
	S-CP.A.3		▪	
	S-CP.A.4		▪	
	S-CP.A.5		▪	
	S-CP.B.6		▪	
	S-CP.B.7		▪	

Mathematics | High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students might learn in order to take advanced courses are included in the Advanced Mathematics Plus and Algebra III courses. The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group**.**

Mathematics | High School—Number and Quantity Conceptual Category

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematics | High School—Algebra Conceptual Category

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Mathematics | High School—Algebra Conceptual Category (continued)

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Mathematics | High School—Functions Conceptual Category

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematics | High School—Modeling Conceptual Category

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

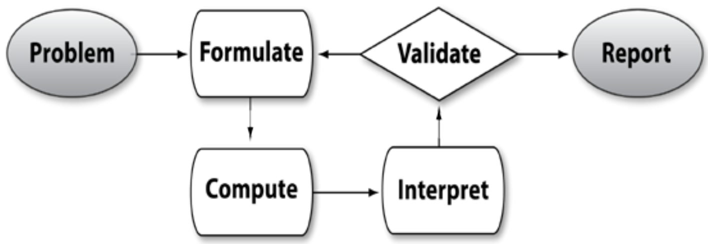
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and

Mathematics | High School—Modeling Conceptual Category (continued)

other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Mathematics | High School—Modeling Conceptual Category (continued)

Modeling Standards. *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).*

Mathematics | High School—Geometry Conceptual Category

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many

Mathematics | High School—Geometry Conceptual Category (continued)

real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematics | High School—Statistics and Probability * Conceptual Category

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematics | High School—Algebra I Course

In Algebra I, a one-credit course, the fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. Instruction should focus on five critical areas: (1) analyze and explain the process of solving equations and inequalities: (2) learn function notation and develop the concepts of domain and range; (3) use regression techniques; (4) create quadratic and exponential expressions; and (5) select from among these functions to model phenomena. Each critical area is described below.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Mathematics | High School—Algebra I Course

(4) In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

(5) In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4-6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (**N-Q**).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (**6.EE.A.3**, **7.EE.A.1**). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (**7.EE.A.2**). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[73]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions. Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.

- Students in grade 8 connected their knowledge about proportional relationships, lines, and linear equations (**8.EE.B.5, 6**). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:
    - The graph of any linear equation in two variables is a line.
    - Any line is the graph of a linear equation in two variables.

[73] See, for example, "Mindful Manipulation," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., **S-ID.B.6**). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.
- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K-8. This expands problem solving from grades K-8 dramatically.

Discussion of Mathematical Practices in Relation to Course Content

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them** (MP.1).
- **Model with mathematics** (MP.4).

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively** (MP.2). This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (**A-CED, F-BF**) and then transforming the models via algebraic calculations (**A-SSE, A-APR, F-IF**) to reveal properties of the problems.
- **Use appropriate tools strategically** (MP.5). Spreadsheets, a function modeling language, graphing tools, and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.
- **Attend to precision** (MP.6). In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (**A-CED, A-REI**) helps students understand the idea in new ways.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

- **Look for and make use of structure** (MP.7). For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$, a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and $z^2 + 5z + 6$, leading to a factorization of the original: $((7x) + 3)((7x) + 2)$ (**A-SSE, A-APR**).
- **Look for and express regularity in repeated reasoning** (MP.8). Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (A-CED). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans.

Fluency Recommendations

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

Content Emphases by Cluster

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

○ D. Use properties of rational and irrational numbers (3)

Quantities*(N-Q)

◘ B. Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)

■ C. Interpret the structure of expressions (1, 2)

◘ D. Write expressions in equivalent forms to solve problems (3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ C. Perform arithmetic operations on polynomials (1)

◘ D. Understand the relationship between zeros and factors of polynomials (3)

Creating Equations* (A-CED)

■ B. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

■ E. Understand solving equations as a process of reasoning and explain the reasoning (1)

■ F. Solve equations and inequalities in one variable (3, 4)

○ G. Solve systems of equations (5, 6)

■ H. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

■ D. Understand the concept of a function and use function notation (1, 2, 3)

■ E. Interpret functions that arise in applications in terms of the context (4, 5, 6)

◘ F. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

◘ C. Build a function that models a relationship between two quantities (1)

○ D. Build new functions from existing functions (3)

Mathematics | High School—Algebra I Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)

- C. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- D. Interpret expressions for functions in terms of the situation they model (5)

Interpreting categorical and quantitative data (S-ID)

- D. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- E. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- F. Interpret linear models (7, 8, 9)

Algebra I

Number and Quantity		
The Real Number System (N-RN)		
Use properties of rational and irrational numbers		**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* c. Interpret parts of an expression, such as terms, factors, and coefficients. d. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* d. Factor a quadratic expression to reveal the zeros of the function it defines. e. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. f. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

Algebra I

Arithmetic with Polynomials and Rational Expressions (A-APR)		
Perform arithmetic operations on polynomials		**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
Understand the relationship between zeros and factors of polynomials		**Supporting**
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
A-REI.4	Solve quadratic equations in one variable. g. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. h. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

Algebra I

	Solve systems of equations	**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
	Represent and solve equations and inequalities graphically	**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
	Functions	
	Interpreting Functions (F-IF)	
	Understand the concept of a function and use function notation	**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	
	Interpret functions that arise in applications in terms of the context	**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	

Algebra I

Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph linear and quadratic functions and show intercepts, maxima, and minima. d. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* d. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* d. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. e. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. f. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	

Algebra I

Interpret expressions for functions in terms of the situation they model		**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* d. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* e. Informally assess the fit of a function by plotting and analyzing residuals. f. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

* Modeling Standards

Algebra I

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Geometry Course

The fundamental purpose of the course in Geometry a one-credit course, is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical areas of this course include (1) building a thorough understanding of translations, reflections, and rotations; (2) developing the understanding of similarity and several theorems; (3) extension of formulas for 2-dimensional and 3-dimensional objects (4) extension of 8th grade geometric concepts of lines; (5) prove basic theorems about circles; and (6) work with experimental and theoretical probability. Each critical area is described below:

(1) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(2) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

(3) Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Mathematics | High School—Geometry Course (continued)

(4) Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

(5) Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

(6) Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Previous Grades or Courses

- Because concepts such as rotation, reflection, and translation were treated in the grade 8 standards mostly in the context of hands-on activities, and with an emphasis on geometric intuition, high school Geometry will put equal weight on precise definitions.

- In grades K-8, students worked with a variety of geometric measures (length, area, volume, angle, surface area, and circumference). In high school Geometry, students apply these component skills in tandem with others in the course of modeling tasks and other substantial applications (**MP.4**).

- The skills that students develop in Algebra I around simplifying and transforming square roots will be useful when solving problems that involve distance or area and that make use the Pythagorean theorem.

- In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (**8.G.B.6–8**). In high school Geometry, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of circles (**G-GPE.A.1**).

- The algebraic techniques developed in Algebra I can be applied to study analytic geometry. Geometric objects can be analyzed by the algebraic equations that give rise to them. Some basic geometric theorems in the Cartesian plane can be proven using algebra.

Discussion of Mathematical Practices in Relation to Course Content

- **Reason abstractly and quantitatively** (**MP.2**). Abstraction is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about *all* isosceles triangles (G-CO.C.9). Quantitative reasoning in geometry involves the real numbers in an essential way: Irrational numbers show up in work with the Pythagorean theorem (G-SRT.C.8), area formulas often depend (subtly and informally) on passing to the limit and real numbers are an essential part of the definition of dilation (G-SRT.A.1). The proper use of units can help students understand the effect of dilation on area and perimeter (N-Q.A.1).

- **Construct viable arguments and critique the reasoning of others** (**MP.3**). While all of high school mathematics should work to help students see the importance and usefulness of deductive arguments, geometry is an ideal arena for developing the skill of creating and presenting proofs (G-CO.C.9.10). One reason is that conjectures about geometric phenomena are often about infinitely many cases at once — for example, *every* angle inscribed in a semicircle is a right

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators (continued)

angle — so that such results cannot be established by checking every case (G-C.A.2).

- **Use appropriate tools strategically** (**MP.5**). Dynamic geometry environments can help students look for invariants in a whole class of geometric constructions, and the constructions in such environments can sometimes lead to an idea behind a proof of a conjecture.

- **Attend to precision** (**MP.6**). Teachers might use the activity of creating definitions as a way to help students see the value of precision. While this is possible in every course, the activity has a particularly visual appeal in geometry. For example, a class can build the definition of *quadrilateral* by starting with a rough idea ("four sides"), gradually refining the idea so that it rules out figures that do not fit the intuitive idea. Another place in geometry where precision is necessary and useful is in the refinement of conjectures so that initial conjectures that are not correct can be salvaged — two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem (G-CO.B.8).

- **Look for and make use of structure** (**MP.7**). Seeing structure in geometric configurations can lead to insights and proofs. This often involves the creation of auxiliary lines not originally part of a given figure. Two classic examples are the construction of a line through a vertex of a triangle parallel to the opposite side as a way to see that the angle measures of a triangle add to 180 degrees and the introduction of a symmetry line in an isosceles triangle to see that the base angles are congruent (G-CO.C.9, 10). Another kind of hidden structure makes use of area as a device to establish results about proportions, such as the important theorem (and its converse) that a line parallel to one side of a triangle divides the other two sides proportionally (G-SRT.B.4).

Fluency Recommendations

G-SRT.B.5	Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.
G-GPE.B.4, 5, 7	Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.

Mathematics | High School—Geometry Course

PARCC Model Content Framework Indicators (continued)

G-CO.D.12 Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◻ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

Congruence (G-CO)

◻ E. Experiment with transformations in the plane (1, 2, 3, 4, 5)

■ F. Understand congruence in terms of rigid motions (6, 7, 8)

■ G. Prove geometric theorems (9, 10, 11)

◻ H. Make geometric constructions (12, 13)

Similarity, Right Triangles, and Trigonometry (G-SRT)

■ D. Understand similarity in terms of similarity transformations (1, 2, 3)

■ E. Prove theorems involving similarity (4, 5)

■ F. Define trigonometric ratios and solve problems involving right triangles (6, 7, 8)

Circles (G-C)

○ C. Understand and apply theorems about circles (1, 2, 3)

○ D. Find arc lengths and areas of sectors of circles (5)

Expressing Geometric Properties with Equations (G-GPE)

○ C. Translate between the geometric description and the equation for a conic section (1)

■ D. Use coordinates to prove simple geometric theorems algebraically (4, 5, 6, 7)

Geometric measurement and dimension (G-GMD)

○ C. Explain volume formulas and use them to solve problems (1, 3)

○ D. Visualize relationships between two-dimensional and three-dimensional objects (4)

Modeling with Geometry (G-MG)

■ B. Apply geometric concepts in modeling situations (1, 2, 3)

Geometry Course

Geometry	
Congruence (G-CO)	
Experiment with transformations in the plane	**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions	**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Prove geometric theorems	**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Geometry Course

Make geometric constructions		**Supporting**
G-CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	
G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Similarity, Right Triangles, and Trigonometry (G-SRT)		
Understand similarity in terms of similarity transformations		**Major**
G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: c. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. d. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
Prove theorems involving similarity		**Major**
G-SRT.4	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Define trigonometric ratios and solve problems involving right triangles		**Major**
G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.	
G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	
Circles (G-C)		
Understand and apply theorems about circles		**Additional**
G-C.1	Prove that all circles are similar.	

Geometry Course

G-C.2	Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
Find arc lengths and areas of sectors of circles		**Additional**
G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
Use coordinates to prove simple geometric theorems algebraically		**Major**
G-GPE.4	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	
G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	
Geometric Measurement and Dimension (G-GMD)		
Explain volume formulas and use them to solve problems		**Additional**
G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	
Visualize relationships between two-dimensional and three-dimensional objects		**Additional**
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Geometry Course

Modeling with Geometry (G-MG)		
Apply geometric concepts in modeling situations		**Major**
G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	
G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	

* Modeling Standards

Geometry Course

Additional Resources:

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Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Algebra II Course

In Algebra II, a one-credit course, students build on their work with linear, quadratic, and exponential functions, to extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with polynomial operations; (2) building connections between geometry and trigonometric ratios; (3) understanding of a variety of function families; and (4)explore statistical data. Each critical area is described below:

(1) Students develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(2) Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

(3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when

Mathematics | High School—Algebra II Course

knowledge of functions, statistics, and geometry is applied in a modeling context.

(4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Previous Grades or Courses

- In Algebra I, students added, subtracted, and multiplied polynomials. In Algebra II, students divide polynomials with remainder, leading to the factor and remainder theorems. This is the underpinning for much of advanced algebra, including the algebra of rational expressions.

- Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (**6.EE.B.5**). This perspective continues throughout Algebra I and Algebra II (**A-REI**).[74] "Reasoned solving" plays a role in Algebra II because the equations students encounter can have extraneous solutions (**A-REI.A.2**).

- In Algebra II, they extend the real numbers to complex numbers, and one effect is that they now have a complete theory of quadratic equations: Every quadratic equation with complex coefficients has (counting multiplicities) two roots in the complex numbers.

- In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (**8.G.B.6–8**). In Geometry, students proved theorems using coordinates (**G-GPE.B.4–7**). In Algebra II, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of conic sections (e.g., **G-GPE.A.1**).

- In Geometry, students began trigonometry through a study of right triangles. In Algebra II, they extend the three basic functions to the entire unit circle.

- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., **S-ID.B.6**). In a modeling context, they might informally fit an exponential function to a set of data, graphing the data and the model function on the same coordinate axes.

Discussion of Mathematical Practices in Relation to Course Content

While all of the mathematical practice standards are important in all three courses, four are especially important in the Algebra II course:

[74] See, for example, "Reasoned Solving," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators (continued)

- **Construct viable arguments and critique the reasoning of others** (**MP.3**). As in geometry, there are central questions in advanced algebra that cannot be answered definitively by checking evidence. There are important results about *all* functions of a certain type — the factor theorem for polynomial functions, for example — and these require general arguments (A-APR.B.2). Deciding whether two functions are equal on an infinite set cannot be settled by looking at tables or graphs; it requires arguments of a different sort (F-IF.C.8).

- **Attend to precision** (**MP.6**). As in the previous two courses, the habit of using precise language is not only a tool for effective communication but also a means for coming to understanding. For example, when investigating loan payments, if students can articulate something like, "What you owe at the end of a month is what you owed at the start of the month, plus $^1/_{12}$ of the yearly interest on that amount, minus the monthly payment," they are well along a path that will let them construct a recursively defined function for calculating loan payments (A-SSE.B.4).

- **Look for and make use of structure** (**MP.7**). The structure theme in Algebra I centered on seeing and using the structure of algebraic expressions. This continues in Algebra II, where students delve deeper into transforming expressions in ways that reveal meaning. The example given in the standards — that $x^4 – y^4$ can be seen as the difference of squares — is typical of this practice. This habit of seeing subexpressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of $x^4 – y^4$ described above can be used to show that the functions $\cos^4 x – \sin^4 x$ and $\cos^2 x – \sin^2 x$ are, in fact, equal (A-SSE.A.2).

  In addition, the standards call for attention to the structural similarities between polynomials and integers (A-APR.A.1). The study of these similarities can be deepened in Algebra II: Like integers, polynomials have a division algorithm, and division of polynomials can be used to understand the factor theorem, transform rational expressions, help solve equations, and factor polynomials.

- **Look for and express regularity in repeated reasoning** (**MP.8**). Algebra II is where students can do a more complete analysis of sequences (F-IF.A.3), especially arithmetic and geometric sequences, and their associated series. Developing recursive formulas for sequences is facilitated by the practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in algebraic symbols (F-BF.A.2). Technology can be a useful tool here: Most Computer Algebra Systems allow one to model recursive function definitions in notation that is close to standard mathematical

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators (continued)

notation. And spreadsheets make natural the process of taking successive differences and running totals (MP.5).

The same thinking — finding and articulating the rhythm in calculations — can help students analyze mortgage payments, and the ability to get a closed form for a geometric series lets them make a complete analysis of this topic. This practice is also a tool for using difference tables to find simple functions that agree with a set of data.

Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount (MP.1). For example, analyzing Heron's formula from geometry lets one connect the zeros of the expression to the degenerate triangles. As in Algebra I, the modeling practice is ubiquitous in Algebra II, enhanced by the inclusion of exponential and logarithmic functions as modeling tools (MP.4). Computer algebra systems provide students with a tool for modeling all kinds of phenomena, experimenting with algebraic objects (e.g., sequences of polynomials), and reducing the computational overhead needed to investigate many classical and useful areas of algebra (MP.5).

Fluency Recommendations

A-APR.D.6 This standard sets an expectation that students will divide polynomials with remainder by inspection in simple cases. For example, one can view the rational expression $\frac{x+4}{x+3}$ as $\frac{x+4}{x+3} = \frac{(x+3)+1}{x+3} = 1 + \frac{1}{x+3}$.

A-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

F-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators (continued)

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments Course emphases are indicated by: ■ Major Content; ◻ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

■ B. Extend the properties of exponents to rational exponents (1, 2)

Quantities* (N-Q)

◻ B. Reason quantitatively and use units to solve problems (2)

The Complex Number System (N-CN)

○ E. Perform arithmetic operations with complex numbers (1, 2)

○ C. Use complex numbers in polynomial identities and equations (7)

Seeing Structure in Expressions (A-SSE)

■ C. Interpret the structure of expressions (2)

■ D. Write expressions in equivalent forms to solve problems (3, 4)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ F. Understand the relationship between zeros and factors of polynomials (2, 3)

○ G. Use polynomial identities to solve problems (4)

◻ H. Rewrite rational expressions (6)

Creating Equations* (A-CED)

◻ A. Create equations that describe numbers or relationships (1)

Reasoning with Equations and Inequalities (A-REI)

■ E. Understand solving equations as a process of reasoning and explain the reasoning (1, 2)

◻ F. Solve equations and inequalities in one variable (4)

○ G. Solve systems of equations (6, 7)

■ H. Represent and solve equations and inequalities graphically (11)

Interpreting Functions (F-IF)

◻ D. Understand the concept of a function and use function notation (3)

■ E. Interpret functions that arise in applications in terms of the context (4, 6)

◻ F. Analyze functions using different representations (7, 8, 9)

Building Functions (F-BF)

■ C. Build a function that models a relationship between two quantities (1, 2)

○ D. Build new functions from existing functions (3, 4a)

Mathematics | High School—Algebra II Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)	
■	C. Construct and compare linear, quadratic, and exponential models and solve problems (2, 4)
●	D. Interpret expressions for functions in terms of the situation they model (5)
Trigonometric Functions (F-TF)	
●	D. Extend the domain of trigonometric functions using the unit circle (1, 2)
●	E. Model periodic phenomena with trigonometric functions (5)
●	F. Prove and apply trigonometric identities (8)
Expressing Geometric Properties with Equations (G-GPE)	
●	B. Translate between the geometric description and the equation for a conic section (2)
Interpreting categorical and quantitative data (S-ID)	
●	C. Summarize, represent, and interpret data on a single count or measurement variable (4)
■	D. Summarize, represent, and interpret data on two categorical and quantitative variables (6)
Making Inferences and Justifying Conclusions (S-IC)	
■	C. Understand and evaluate random processes underlying statistical experiments (1, 2)
■	D. Make inferences and justify conclusions from sample surveys, experiments and observational studies (3, 4, 5, 6)
Conditional Probability and the Rules of Probability (S-CP)	
●	C. Understand independence and conditional probability and use them to interpret data (1, 2, 3, 4, 5)
●	D. Use the rules of probability to compute probabilities of compound events in a uniform probability model (6, 7)

Algebra II

<table>
<tr><th colspan="3">Number and Quantity</th></tr>
<tr><th colspan="3">The Real Number System (N-RN)</th></tr>
<tr><td colspan="2">Extend the properties of exponents to rational exponents</td><td>Major</td></tr>
<tr><td>N-RN.1</td><td colspan="2">Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{(1/3)\,3}$ to hold, so $[5^{1/3}]^3$ must equal 5.</td></tr>
<tr><td>N-RN.2</td><td colspan="2">Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td></tr>
<tr><th colspan="3">Quantities (N-Q) *</th></tr>
<tr><td colspan="2">Reason quantitatively and use units to solve problems</td><td>Supporting</td></tr>
<tr><td>N-Q.2</td><td colspan="2">Define appropriate quantities for the purpose of descriptive modeling.*</td></tr>
<tr><th colspan="3">The Complex Number System (N-CN)</th></tr>
<tr><td colspan="2">Perform arithmetic operations with complex numbers</td><td>Additional</td></tr>
<tr><td>N-CN.1</td><td colspan="2">Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.</td></tr>
<tr><td>N-CN.2</td><td colspan="2">Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</td></tr>
<tr><td colspan="2">Use complex numbers in polynomial identities and equations</td><td>Additional</td></tr>
<tr><td>N-CN.7</td><td colspan="2">Solve quadratic equations with real coefficients that have complex solutions.</td></tr>
<tr><th colspan="3">Algebra</th></tr>
<tr><th colspan="3">Seeing Structure in Expressions (A-SSE)</th></tr>
<tr><td colspan="2">Interpret the structure of expressions</td><td>Major</td></tr>
<tr><td>A-SSE.2</td><td colspan="2">Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.</td></tr>
<tr><td colspan="2">Write expressions in equivalent forms to solve problems</td><td>Major</td></tr>
<tr><td>A-SSE.3</td><td colspan="2">Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
d. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</td></tr>
</table>

Algebra II

A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Understand the relationship between zeros and factors of polynomials		**Major**
A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Use polynomial identities to solve problems		**Additional**
A-APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*	
Rewrite rational expressions		**Supporting**
A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Supporting**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Solve equations and inequalities in one variable		**Supporting**
A-REI.4	Solve quadratic equations in one variable. d. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

Algebra II

Solve systems of equations		**Additional**
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
Functions		
Interpreting Functions (F-IF)		
Understand the concept of a function and use function notation		**Supporting**
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* e. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. f. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

Algebra II

F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. c. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth and decay.*
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Building Functions (F-BF)	
Build a function that models a relationship between two quantities	**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* c. Determine an explicit expression, a recursive process, or steps for calculation from a context. d. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
Build new functions from existing functions	**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-BF.4	Find inverse functions. b. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
Linear, Quadratic, and Exponential Models (F-LE) *	
Construct and compare linear, quadratic, and exponential models and solve problems	**Supporting**
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
Interpret expressions for functions in terms of the situation they model	**Additional**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*

Algebra II

Trigonometric Functions (F-TF)		
Extend the domain of trigonometric functions using the unit circle		**Additional**
F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
Model periodic phenomena with trigonometric functions		**Additional**
F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	
Prove and apply trigonometric identities		**Additional**
F-TF.8	Prove the Pythagorean identity $\sin(\Theta)^2 + \cos(\Theta)^2 = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$, given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle.	

Geometry

Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.2	Derive the equation of a parabola given a focus and directrix.	

Statistics and Probability*

Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Additional**
S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* b. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	

Algebra II

Making Inferences and Justifying Conclusions (S-IC)		
Understand and evaluate random processes underlying statistical experiments		**Supporting**
S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*	
S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies		**Major**
S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*	
S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*	
S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*	
S-IC.6	Evaluate reports based on data.*	
Conditional Probability and the Rules of Probability (S-CP)		
Understand independence and conditional probability and use them to interpret data		**Additional**
S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).*	
S-CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	
S-CP.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	
S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	
S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	

Algebra II

Use the rules of probability to compute probabilities of compound events in a uniform probability model		**Additional**
S-CP.6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.*	
S-CP.7	Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.*	

* Modeling Standards

Algebra II

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics I Course

The fundamental purpose of Integrated Mathematics I, a one-credit course, is to formalize and extend the mathematics that students learned in the middle grades. The critical areas deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Integrated Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final critical area in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical focus areas of this course include (1)working with quantities to model and analyze situations; (2) exploring sequences and their relationships to functions; (3) working and translating between the various forms of linear equations and inequalities; (4) fitting data to a particular model; (5) establishing triangle congruency; and (6) verifying geometric relationships. Each critical area is described below:

(1) By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first critical focus area, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that,
depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and

Mathematics | High School—Integrated Mathematics I Course (continued)

solve systems of linear equations in two variables. This critical area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

(4) This critical area builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

(5) In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this area, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(6) Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Grades K–8

- Students build on previous work with solving linear equations and systems of linear equations in two ways: (a) They extend to more formal solution methods, including attending to the structure of linear expressions, and (b) they solve linear inequalities.

- Students formalize their understanding of the definition of a function, particularly their understanding of linear functions, emphasizing the structure of linear expressions. Students also begin to work on exponential functions, comparing them to linear functions.

- Work with congruence and similarity motions that started in grades 6-8 progresses. Students also consider sufficient conditions for congruence of triangles.

- Work with the bivariate data and scatter plots in grades 6-8 is extended to working *with lines of best fit.*

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (**MP.4**) should be a particular focus as students see the purpose and meaning for working with linear and exponential equations and functions.

- **Using appropriate tools strategically** (**MP.5**) is also important as students explore those models in a variety of ways, including with technology. For example, students might be given a set of data points and experiment with graphing a line that fits the data.

- As Mathematics I continues to develop a foundation for more formal reasoning, students should engage in the practice of **constructing viable arguments and critiquing the reasoning of others** (**MP.3**).

Fluency Recommendations

A/G High school students should become fluent in solving characteristic problems involving the analytic geometry of lines, such as finding the equation of a line given a point and a slope. This fluency can support students in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators (continued)

G High school students should become fluent in using geometric transformation to represent the relationships among geometric objects. This fluency provides a powerful tool for visualizing relationships, as well as a foundation for exploring ideas both within geometry (e.g., symmetry) and outside of geometry (e.g., transformations of graphs).

S Students should be able to create a visual representation of a data set that is useful in understanding possible relationships among variables.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content. Not all CCSSM standards in a listed domain or cluster are assessed.

Quantities* (N-Q)

□ B. Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)

■ C. Interpret the structure of expressions (1)

□ D. Write expressions in equivalent forms to solve problems (3)

Creating Equations* (A-CED)

■ B. Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

■ B. Solve equations and inequalities in one variable (3)

○ C. Solve systems of equations (5, 6)

■ D. Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

■ D. Understand the concept of a function and use function notation (1, 2, 3)

■ E. Interpret functions that arise in applications in terms of the context (4, 5, 6)

□ F. Analyze functions using different representations (7, 9)

Building Functions (F-BF)

■ B. Build a function that models a relationship between two quantities (1, 2)

Mathematics | High School—Integrated Mathematics I Course

PARCC Model Content Framework Indicators (continued)

Linear, Quadratic, and Exponential Models* (F-LE)

- A. Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- B. Interpret expressions for functions in terms of the situation they model (5)

Congruence (G-CO)

- A. Experiment with transformations in the plane (1, 2, 3, 4, 5)
- B. Understand congruence in terms of rigid motions (6, 7, 8)
- C. Prove geometric theorems (9, 10, 11)

Interpreting categorical and quantitative data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- C. Interpret linear models (7, 8, 9)

Integrated Mathematics I

Number and Quantity

Quantities (N-Q) *

Reason quantitatively and use units to solve problems		**Supporting**
N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*	
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	

Algebra

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions		**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* d. Interpret parts of an expression, such as terms, factors, and coefficients. e. Interpret complicated expressions by viewing one or more of their parts as a single entity*. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	
Write expressions in equivalent forms to solve problems		**Supporting**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* f. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

Creating Equations (A-CED) *

Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	

Integrated Mathematics I

Reasoning with Equations and Inequalities (A-REI)		
Solve equations and inequalities in one variable		**Major**
A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
Solve systems of equations		**Additional**
A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Functions

Interpreting Functions (F-IF)		
Understand the concept of a function and use function notation		**Major**
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	

Integrated Mathematics I

F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
	Analyze functions using different representations	**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
	Build a function that models a relationship between two quantities	**Major**
F-BF.1	Write a function that describes a relationship between two quantities.* b. Determine an explicit expression, a recursive process, or steps for calculation from a context.	
F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Linear, Quadratic, and Exponential Models (F-LE) *		
	Construct and compare linear, quadratic, and exponential models and solve problems	**Supporting**
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.* d. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. e. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. f. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*	
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*	
	Interpret expressions for functions in terms of the situation they model	**Supporting**
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*	

Integrated Mathematics I

Geometry

Congruence (G-CO)

Experiment with transformations in the plane		**Supporting**
G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
Understand congruence in terms of rigid motions		**Major**
G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Prove geometric theorems		**Major**
G-CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	
G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	
G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	

Integrated Mathematics I

Statistics and Probability *

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable		**Supporting**
S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).*	
S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*	
S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Additional**
S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* c. Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models		**Major**
S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*	
S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	
S-ID.9	Distinguish between correlation and causation.*	

* Modeling Standards

Integrated Mathematics I

Additional Resources:

PARCC Model Content Frameworks: http://www.parcconline.org/parcc-model-content-frameworks

PARCC Calculator and Reference Sheet Policies: http://www.parcconline.org/assessment-administration-guidance

PARCC Sample Assessment Items: http://www.parcconline.org/samples/item-task-prototypes

PARCC Performance Level Descriptors (PLDs): http://www.parcconline.org/math-plds

PARCC Assessment Blueprints/Evidence Tables: http://www.parcconline.org/assessment-blueprints-test-specs

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics II Course

The focus of Integrated Mathematics II, a one-credit course, is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Integrated Mathematics I as organized into 6 critical areas. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, bring more depth to the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The six critical areas of this course include (1) exploring the distinction between rational and irrational numbers; (2) expending expertise of functions into piece-wise functions and quadratics; (3) focusing on the structure of expressions; (4) exploring compound events; (5) building a formal understanding of similarity; and (6) proving basic theorems about circles. Each critical area is described below:

> (1) Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.
>
> (2) Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.
> When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Mathematics | High School—Integrated Mathematics II Course (continued)

(3) Students begin this critical area by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

(4) Students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

(5) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

(6) In this area students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles is also explored.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Mathematics I

- Students extend their previous work with linear and exponential expressions, equations, systems of equations, and inequalities to quadratic relationships.
- A parallel extension occurs from linear and exponential functions to quadratic functions, where students also begin to analyze functions in terms of transformations.
- Building on their work with transformations, students produce increasingly formal arguments about geometric relationships, particularly around notions of similarity.

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (**MP.4**) should be a particular focus as students see the purpose and meaning for working with quadratic equations and functions, including **using appropriate tools strategically** (**MP.5**).
- As students explore a variety of ways to represent quadratic expressions, they should **look for and make use of structure** (**MP.7**).
- As their ability to create and use formal mathematical arguments grows, increased emphasis is placed on students' ability to **attend to precision** (**MP.6**), as well as to **construct viable arguments and critique the reasoning of others** (**MP.3**).

Fluency Recommendations

F/S Fluency in graphing functions (including linear, quadratic, and exponential) and interpreting key features of the graphs in terms of their function rules and a table of value, as well as recognizing a relationship (including a relationship within a data set), fits one of those classes. This forms a critical base for seeing the value and purpose of mathematics, as well as for further study in mathematics.

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

G-SRT.B.5 Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in geometric modeling.

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators (continued)

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

■ A. Extend the properties of exponents to rational exponents (1, 2)

○ B. Use properties of rational and irrational numbers (3)

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (<u>2</u>)

The Complex Number System (N-CN)

○ A. Perform arithmetic operations with complex numbers (1, 2)

○ C. Use complex numbers in polynomial identities and equations (7)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (<u>1</u>, <u>2</u>)

■ B. Write expressions in equivalent forms to solve problems (<u>3</u>)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ A. Perform arithmetic operations on polynomials (1)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (<u>1</u>, <u>2</u>, <u>4</u>)

Reasoning with Equations and Inequalities (A-REI)

■ A. Understand solving equations as a process of reasoning and explain the reasoning (<u>1</u>)

■ B. Solve equations and inequalities in one variable (4)

○ C. Solve systems of equations (7)

Interpreting Functions (F-IF)

■ B. Interpret functions that arise in applications in terms of the context (<u>4</u>, <u>5</u>, <u>6</u>)

◘ C. Analyze functions using different representations (<u>7</u>, 8, <u>9</u>)

Building Functions (F-BF)

◘ A. Build a function that models a relationship between two quantities (<u>1</u>)

○ B. Build new functions from existing functions (<u>3</u>)

Mathematics | High School—Integrated Mathematics II Course

PARCC Model Content Framework Indicators (continued)

Similarity, Right Triangles, and Trigonometry (G-SRT)

- A. Understand similarity in terms of similarity transformations (1, 2, 3)
- B. Prove theorems using similarity (4, 5)
- C. Define trigonometric ratios and solve problems involving right triangles (6, 7, 8)

Geometric Measurement and Dimension (G-GMD)

- A. Explain volume formulas and use them to solve problems (1, 3)

Interpreting categorical and Quantitative Data (S-ID)

- B. Summarize, represent, and interpret data on two categorical and quantitative variables (6)

Conditional Probability and Rules of Probability (S-CP)

- A. Understand independence and conditional probability and use them to interpret data (1, 2, 3, 4, 5)
- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model (6, 7)

Integrated Mathematics II

Number and Quantity		
The Real Number System (N-RN)		
	Extend the properties of exponents to rational exponents	**Major**
N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{(1/3)\,3}$ to hold, so $[5^{1/3}]^3$ must equal 5.*	
N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
	Use properties of rational and irrational numbers	**Additional**
N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
Quantities (N-Q) *		
	Reason quantitatively and use units to solve problems	**Supporting**
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
The Complex Number System (N-CN)		
	Perform arithmetic operations with complex numbers	**Additional**
N-CN.1	Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.	
N-CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
	Use complex numbers in polynomial identities and equations	**Additional**
N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.	
Algebra		
Seeing Structure in Expressions (A-SSE)		
	Interpret the structure of expressions	**Major**
A-SSE.1	Interpret expressions that represent a quantity in terms of its context.* b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	

Integrated Mathematics II

A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	
	Write expressions in equivalent forms to solve problems	**Major**
A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* c. Factor a quadratic expression to reveal the zeros of the function it defines. d. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
	Arithmetic with Polynomials and Rational Expressions (A-APR)	
	Perform arithmetic operations on polynomials	**Major**
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
	Creating Equations (A-CED) *	
	Create equations that describe numbers or relationships	**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**	
	Reasoning with Equations and Inequalities (A-REI)	
	Understand solving equations as a process of reasoning and explain the reasoning	**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
	Solve equations and inequalities in one variable	**Major**
A-REI.4	Solve quadratic equations in one variable. c. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. d. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	

Integrated Mathematics II

Solve systems of equations		**Additional**
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	
Functions		
Interpreting Functions (F-IF)		
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph linear and quadratic functions and show intercepts, maxima, and minima. d. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. c. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. d. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.*	
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	

Integrated Mathematics II

Building Functions (F-BF)

Build a function that models a relationship between two quantities		**Supporting**
F-BF.1	Write a function that describes a relationship between two quantities.* c. Determine an explicit expression, a recursive process, or steps for calculation from a context. d. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	

Geometry

Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations		**Major**
G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: c. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. d. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
Prove theorems using similarity		**Major**
G-SRT.4	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Define trigonometric ratios and solve problems involving right triangles		**Major**
G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.	

Integrated Mathematics II

G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	
Geometric Measurement and Dimension (G-GMD)		
	Explain volume formulas and use them to solve problems	**Additional**
G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
	Summarize, represent, and interpret data on two categorical and quantitative variables	**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* c. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* d. Informally assess the fit of a function by plotting and analyzing residuals.	
Conditional Probability and the Rules of Probability (S-CP)		
	Understand independence and conditional probability and use them to interpret data	**Additional**
S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*	
S-CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	
S-CP.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	
S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	
S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	

Integrated Mathematics II

Use the rules of probability to compute probabilities of compound events in a uniform probability model		Additional
S-CP.6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.*	
S-CP.7	Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.*	

* Modeling Standards

Integrated Mathematics II

Additional Resources:

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Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Integrated Mathematics III Course

It is in Mathematics III, a one-credit course, that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with statistics and probability; (2) culminating work with the Fundamental Theorem of Algebra; (3) understanding of periodic phenomena; and (4) exploring function fitting. Each critical area is described below:

(1) In this area, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.
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(2) This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The area culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

(3) Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2,or infinitely many triangles. This discussion of general triangles open up the idea

Mathematics | High School—Integrated Mathematics III Course (continued)

of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

(4) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators

Examples of Key Advances from Mathematics II

- Students begin to see polynomials as a system that has mathematical coherence, not just as a set of expressions of a specific type. An analogy to the integers can be made (including operations, factoring, etc.). Subsequently, polynomials can be extended to rational expressions, analogous to the rational numbers.
- The understandings that students have developed with linear, exponential and quadratic functions are extended to considering a much broader range of classes of functions.
- In statistics, students begin to look at the role of randomization in statistical design.

Discussion of Mathematical Practices in Relation to Course Content

- **Modeling with mathematics** (MP.4) continues to be a particular focus as students see a broader range of functions, including **using appropriate tools strategically** (**MP.5**).
- **Constructing viable arguments and critiquing the reasoning of others** (MP.3) continues to be a focus, as does **attention to precision** (MP.6), because students are expected to provide increasingly precise arguments.
- As students continue to explore a range of algebraic expressions, including polynomials, they should **look for and make use of structure** (MP.7).
- Finally, as students solidify the tools they need to continue their study of mathematics, a focus on **making sense of problems and persevering in solving them** (MP.1) is an essential component for their future success.

Fluency Recommendations

A/F Students should look at algebraic manipulation as a meaningful enterprise, in which they seek to understand the structure of an expression or equation and use properties to transform it into forms that provide useful information (e.g., features of a function or solutions to an equation). This perspective will help students continue to usefully apply their mathematical knowledge in a range of situations, whether their continued study leads them toward college or career readiness.

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators (continued)

M	Seeing mathematics as a tool to model real-world situations should be an underlying perspective in everything students do, including writing algebraic expressions, creating functions, creating geometric models, and understanding statistical relationships. This perspective will help students appreciate the importance of mathematics as they continue their study of it.
N-Q	In particular, students should recognize that much of mathematics is concerned with understanding quantities and their relationships. They should pick appropriate units for quantities being modeled, using them as a guide to understand a situation, and be attentive to the level of accuracy that is reported in a solution.
F-BF.B.3	Students should understand the effects of parameter changes and be able to apply them to create a rule modeling the function.

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., <u>1</u>) indicate standards eligible for assessment on two or more end-of-course assessments. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

Quantities* (N-Q)

◘ A. Reason quantitatively and use units to solve problems (<u>2</u>)

Seeing Structure in Expressions (A-SSE)

■ A. Interpret the structure of expressions (<u>2</u>)

■ B. Write expressions in equivalent forms to solve problems (4)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ B. Understand the relationship between zeros and factors of polynomials (2, 3)

○ C. Use polynomial identities to solve problems (4)

○ D. Rewrite rational expressions (6)

Creating Equations* (A-CED)

■ A. Create equations that describe numbers or relationships (<u>1</u>, <u>2</u>)

Mathematics | High School—Integrated Mathematics III Course

PARCC Model Content Framework Indicators (continued)

Reasoning with Equations and Inequalities (A-REI)

- A. Understand solving equations as a process of reasoning and explain the reasoning (1, 2)
- D. Represent and solve equations and inequalities graphically (11)

Interpreting Functions (F-IF)

- B. Interpret functions that arise in applications in terms of the context (4, 6)
- C. Analyze functions using different representations (7, 9)

Building Functions (F-BF)

- B. Build new functions from existing functions (3, 4a)

Linear, Quadratic, and Exponential Models * (F-LE)

- A. Construct and compare linear, quadratic, and exponential models and solve problems (4)

Trigonometric Functions (F-TF)

- A. Extend the domain of trigonometric functions using the unit circle (1, 2)
- B. Model periodic phenomena with trigonometric functions (5)
- C. Prove and apply trigonometric identities (8)

Congruence (G-CO)

- D. Make geometric constructions (12, 13)

Circles (G-C)

- A. Understand and apply theorems about circles (1, 2, 3)
- B. Find arc lengths and areas of sectors of circles (5)

Expressing Geometric Properties with Equations (G-GPE)

- A. Translate between the geometric description and the equation for a conic section (1, 2)
- B. Use coordinates to prove simple geometric theorems algebraically (4, 5, 6, 7)

Geometric Measurement and Dimension (G-GMD)

- B. Visualize relationships between two-dimensional and three-dimensional objects (4)

Modeling with Geometry (G-MG)

- A. Apply geometric concepts in modeling situations (1, 2, 3)

Interpreting Categorical and Quantitative Data (S-ID)

- A. Summarize, represent, and interpret data on a single count or measurement variable (4)
- B. Summarize, represent, and interpret data on two categorical and quantitative variables (6)

Making Inferences and Justifying Conclusions (S-IC)

- A. Understand and evaluate random processes underlying statistical experiments (1, 2)
- B. Make inferences and justify conclusions from sample surveys, experiments and observational studies (3, 4, 5, 6)

Integrated Mathematics III

Number and Quantity		
Quantities (N-Q) *		
Reason quantitatively and use units to solve problems		**Supporting**
N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*	
Algebra		
Seeing Structure in Expressions (A-SSE)		
Interpret the structure of expressions		**Major**
A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	
Write expressions in equivalent forms to solve problems		**Major**
A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**	
Arithmetic with Polynomials and Rational Expressions (A-APR)		
Understand the relationship between zeros and factors of polynomials		**Major**
A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Use polynomial identities to solve problems		**Additional**
A-APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	
Rewrite rational expressions		**Additional**
A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	

Integrated Mathematics III

Creating Equations (A-CED) *		
Create equations that describe numbers or relationships		**Major**
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**	
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
Reasoning with Equations and Inequalities (A-REI)		
Understand solving equations as a process of reasoning and explain the reasoning		**Major**
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Represent and solve equations and inequalities graphically		**Major**
A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
Functions		
Interpreting Functions (F-IF)		
Interpret functions that arise in applications in terms of the context		**Major**
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations		**Supporting**
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

Integrated Mathematics III

F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	
Building Functions (F-BF)		
Build new functions from existing functions		**Additional**
F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\, f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	
F-BF.4	Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	
Linear, Quadratic, and Exponential Models (F-LE) *		
Construct and compare linear, quadratic, and exponential models and solve problems		**Supporting**
F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*	
Trigonometric Functions (F-TF)		
Extend the domain of trigonometric functions using the unit circle		**Additional**
F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
Model periodic phenomena with trigonometric functions		**Additional**
F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	
Prove and apply trigonometric identities		**Additional**
F-TF.8	Prove the Pythagorean identity $\sin(\Theta)^2 + \cos(\Theta)^2 = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$, given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle.	

Integrated Mathematics III

Geometry		
Congruence (G-CO)		
Make geometric constructions		**Supporting**
G-CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	
G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Circles (G-C)		
Understand and apply theorems about circles		**Additional**
G-C.1	Prove that all circles are similar.	
G-C.2	Identify and describe relationships among inscribed angles, radii, and chords*. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
Find arc lengths and areas of sectors of circles		**Additional**
G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Expressing Geometric Properties with Equations (G-GPE)		
Translate between the geometric description and the equation for a conic section		**Additional**
G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
G-GPE.2	Derive the equation of a parabola given a focus and directrix.	
Use coordinates to prove simple geometric theorems algebraically		**Major**
G-GPE.4	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	
G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	

Integrated Mathematics III

G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	
Geometric Measurement and Dimension (G-GMD)		
Visualize relationships between two-dimensional and three-dimensional objects		**Additional**
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	
Modeling with Geometry (G-MG)		
Apply geometric concepts in modeling situations		**Major**
G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	
G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	
G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	
Statistics and Probability *		
Interpreting Categorical and Quantitative Data (S-ID)		
Summarize, represent, and interpret data on a single count or measurement variable		**Supporting**
S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*	
Summarize, represent, and interpret data on two categorical and quantitative variables		**Supporting**
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* c. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* d. Informally assess the fit of a function by plotting and analyzing residuals.	
Making Inferences and Justifying Conclusions (S-IC)		
Understand and evaluate random processes underlying statistical experiments		**Supporting**
S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*	

Integrated Mathematics III

S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies		**Major**
S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*	
S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*	
S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*	
S-IC.6	Evaluate reports based on data.*	

* Modeling Standards

Integrated Mathematics III

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Mathematics | High School—Advanced Mathematics Plus Course

Advanced Mathematics Plus, a one-credit course, specifies the mathematics that students should study in order to be college and career ready. The Advanced Mathematics Plus Course includes additional mathematics from the *Common Core State Standards for Mathematics* indicated by a (+). These "plus standards" will help students in advanced courses such as Calculus. This course was designed to be a fourth year Common Core math course. Throughout the duration of this course, teachers should make every effort to ensure the Standards for Mathematical Practice are addressed.

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Advanced Mathematics Plus

Number and Quantity	
The Complex Number System (N-CN)	
Perform arithmetic operations with complex numbers	
N-CN.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane	
N-CN.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N-CN.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.*
N-CN.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Use complex numbers in polynomial identities and equations	
N-CN.8	Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
N-CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Vector and Matrix Quantities (N-VM)	
Represent and model with vector quantities	
N-VM.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $\lvert\mathbf{v}\rvert$, $\lVert\mathbf{v}\rVert$, v).
N-VM.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N-VM.3	Solve problems involving velocity and other quantities that can be represented by vectors.

Advanced Mathematics Plus

Perform operations on vectors	
N-VM.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N-VM.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lvert\lvert c\boldsymbol{v}\rvert\rvert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
Perform operations on matrices and use matrices in applications	
N-VM.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N-VM.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N-VM.8	Add, subtract, and multiply matrices of appropriate dimensions.
N-VM.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM.12	Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
Algebra	
Arithmetic with Polynomials and Rational Expressions (A-APR)	
Use polynomial identities to solve problems	
A-APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Advanced Mathematics Plus

Rewrite rational expressions	
A-APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Reasoning with Equations and Inequalities (A-REI)	
Solve systems of equations	
A-REI.8	Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
Functions	
Interpreting Functions (F-IF)	
Analyze functions using different representations	
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* d. Graph rational functions, indentifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
Building Functions (F-BF)	
Build a function that models a relationship between two quantities	
F-BF.1	Write a function that describes a relationship between two quantities. * c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
Build new functions from existing functions	
F-BF.4	Find inverse functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.
F-BF.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Advanced Mathematics Plus

Trigonometric Functions (F-TF)	
Extend the domain of trigonometric functions using the unit circle	
F-TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
F-TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions	
F-TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *
Prove and apply trigonometric identities	
F-TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Geometry

Similarity, Right Triangles, and Trigonometry (G-SRT)	
Apply trigonometry to general triangles	
G-SRT.9	Derive the formula $A = ½ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G-SRT.10	Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Circles (G-C)	
Understand and apply theorems about circles	
G-C.4	Construct a tangent line from a point outside a given circle to the circle.
Expressing Geometric Properties with Equations (G-GPE)	
Translate between the geometric description and the equation for a conic section	

Advanced Mathematics Plus

G-GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Geometric Measurement and Dimension (G-GMD)	
Explain volume formulas and use them to solve problems	
G-GMD.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Statistics and Probability *	
Conditional Probability and the Rules of Probability (S-CP)	
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
S-CP.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.*
S-CP.9	Use permutations and combinations to compute probabilities of compound events and solve problems.*
Using Probability to Make Decisions (S-MD)	
Calculate expected values and use them to solve problems	
S-MD.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.*
S-MD.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.*
S-MD.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.**
S-MD.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?**

Advanced Mathematics Plus

Use probability to evaluate outcomes of decisions	
S-MD.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. * a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant*. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.**
S-MD.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
S-MD.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

* Modeling Standards

Advanced Mathematics Plus

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Algebra III Course

Algebra III, a one-credit course, includes content standards from the *2007 Mississippi Mathematics Framework Revised* Pre-Calculus course and the Common Core State Standards for Mathematics, and covers those skills and objectives necessary for success in courses higher than Algebra II and Integrated Mathematics III. Topics of study include sequences and series, functions, and higher order polynomials. Polynomial functions provide the context for higher-order investigations. Topics are addressed from a numeric, graphical, and analytical perspective. Technology is to be used to enhance presentation and understanding of concepts. The instructional approach should provide opportunities for students to work together collaboratively and cooperatively as they solve routine and non-routine problems. Communication strategies should include reading, writing, speaking, and critical listening as students present and evaluate mathematical arguments, proofs, and explanations about their reasoning. Algebra III is typically taken by students who have successfully completed Algebra II and Geometry.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

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Algebra III

Number and Quantity

Explore and illustrate the characteristics and operations connecting sequences and series	
1	Express sequences and series using recursive and explicit formulas.
2	Evaluate and apply formulas for arithmetic and geometric sequences and series.
3	Calculate limits based on convergent and divergent series.
4	Evaluate and apply infinite geometric series.
5	Extend the meaning of exponents to include rational numbers.
6	Simplify expressions with fractional exponents to include converting from radicals.
7	Factor algebraic expressions containing fractional exponents.

Algebra

Analyze and manipulate functions	
8	Determine characteristics of graphs of parent functions (domain/range, increasing/decreasing intervals, intercepts, symmetry, end behavior, and asymptotic behavior).
9	Determine the end behavior of polynomial functions.
Use polynomial identities to solve problems	
10	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
11	Verify the Binomial Theorem by mathematical induction or by a combinatorial argument.
12	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
13	Write rational expressions in simplest form. (*For* example $\frac{x^3 - x^2 - x + 1}{x^3 + x^2 - x - 1} = \frac{x-1}{x+1}$) .
14	Decompose a rational function into partial fractions.
15	Determine asymptotes and holes of rational functions, explain how each was found, and relate these behaviors to continuity.
Perform operations on expressions, equations, inequalities and polynomials	
16	Add, subtract, multiply and divide rational expressions.
17	Solve polynomial and rational inequalities. Relate results to the behavior of the graphs.

Algebra III

18	Find the composite of two given functions and find the inverse of a given function. Extend this concept to discuss the identity function f(x) = x.
19	Simplify complex algebraic fractions (with/without variable expressions and integer exponents) to include expressing $\frac{f(x+h)-f(x)}{h}$ as a single simplified fraction when f(x) = $\frac{1}{1-x}$ for example.
20	Find the possible rational roots using the Rational Root Theorem.
21	Find the zeros of polynomial functions by synthetic division and the Factor Theorem.
22	Graph and solve quadratic inequalities.

Functions

	Analyze functions using different representations
23	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
24	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
	Build a function that models a relationship between two quantities
25	Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
	Build new functions from existing functions
26	Verify by composition that one function is the inverse of another.
27	Read values of an inverse function from a graph or a table, given that the function has an inverse.
28	Produce an invertible function from a non-invertible function by restricting the domain.
29	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
	Extend the domain of trigonometric functions using the unit circle
30	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
31	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Algebra III

	Model periodic phenomena with trigonometric functions
32	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
33	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
	Prove and apply trigonometric identities
34	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
	Geometry
	Recognize, sketch, and transform graphs of functions
35	Graph piecewise defined functions and determine continuity or discontinuities.
36	Describe the attributes of graphs and the general equations of parent functions (linear, quadratic, cubic, absolute value, rational, exponential, logarithmic, square root, cube root, and greatest integer).
37	Explain the effects of changing the parameters in transformations of functions.
38	Predict the shapes of graphs of exponential, logarithmic, rational, and piece-wise functions, and verify the prediction with and without technology.
39	Relate symmetry of the behavior of even and odd functions.
	Apply trigonometry to general triangles
40	Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
41	Prove the Laws of Sines and Cosines and use them to solve problems.
42	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
	Statistics and Probability
	Explore and apply fundamental principles of probability.
43	Analyze expressions in summation and factorial notation to solve problems.
44	Prove statements using mathematical induction.

Algebra III

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Calculus Course

Calculus, a one-credit course, includes content standards from the *2007 Mississippi Mathematics Framework Revised.* This course focuses on the mathematics of change. The major focus is on differential and integral calculus. The use of graphing calculators and other technologies are major components of the course. The instructional approach should provide opportunities for students to work together collaboratively and cooperatively as they solve routine and non-routine problems. Communication strategies should include reading, writing, speaking, and critical listening as students present and evaluate mathematical arguments, proofs, and explanations about their reasoning. This one-credit course is designed for the student who has been successful in Algebra II, Integrated Mathematics III, or Algebra III.

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Calculus

Number and Quantity

Compute and determine the reasonableness of results in mathematical and real world situations	
1	Estimate limits from graphs or tables.
2	Estimate numerical derivatives from graphs or tables of data.
3	Prove statements using mathematical induction.

Algebra

Demonstrate basic knowledge of functions, including their behavior and characteristics	
4	Predict and explain the characteristics and behavior of functions and their graphs (domain, range, increasing/decreasing intervals, intercepts, symmetry, and end behavior).
5	Investigate, describe, and determine asymptotic behavior using tables, graphs, and analytical methods
6	Determine and justify the continuity and discontinuity of functions
Evaluate limits and communicate an understanding of the limiting process	
7	Solve mathematical situations and application problems involving or using derivatives, including exponential, logarithmic, and trigonometric functions.
8	Calculate limits using algebraic methods.
9	Verify the behavior and direction of non-determinable limits.
Use the definition and formal rules of differentiation to compute derivatives	
10	State and apply the formal definition of a derivative.
11	Apply differentiation rules to sums, products, quotients, and powers of functions.
12	Use the chain rule and implicit differentiation.
13	Describe the relationship between differentiability and continuity.
Apply derivatives to find solutions in a variety of situations	
15	Define a derivative and explain the purpose/utility of the derivative.
16	Apply the derivative as a rate of change in varied contexts, including velocity, speed, and acceleration.
17	Apply the derivative to find tangent lines and normal lines to given curves at given points.
18	Predict and explain the relationships between functions and their derivatives.
19	Model rates of change to solve related rate problems.
20	Solve optimization problems.

Calculus

Employ various integration properties and techniques to evaluate integrals	
21	State and apply the First and Second Fundamental Theorem of Calculus.
22	Apply the power rule and u-substitution to evaluate indefinite integrals.
Geometry	
Use geometric concepts to gain insights into, answer questions about, and graph various implications of differentiation	
23	Demonstrate and explain the differences between average and instantaneous rates of change.
24	Apply differentiation techniques to curve sketching
25	Apply Rolle's Theorem and the Mean Value Theorem and their geometric consequences.
26	Identify and apply local linear approximations.
27	Analyze curves with attention to non-decreasing functions (monotonicity) and concavity.
Statistics and Probability	
Adapt integration methods to model situations to problems	
28	Apply integration to solve problems of area.
29	Utilize integrals to model and find solutions to real-world problems such as calculating displacement and total distance traveled.
Apply appropriate techniques, tools, and formulas to determine values for the definite integral	
30	Interpret the concept of definite integral as a limit of Riemann sums over equal subdivisions.

Calculus

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—SREB Math Ready Course

The Southern Region Education Board (SREB) Math Ready Course, a one-credit course, is designed to assist students who are in need of a fourth year mathematics preparatory course prior to entering college. This course is best suited for students who have not mastered skills needed for Advanced Placement courses. The course is built with rigor, innovative instructional strategies, and a concentration on contextual learning that departs from procedural memorization and focuses on engaging the students in a real-world context.. In short, this course targets students with weaknesses and college-ready skill gaps and re-educates them in new ways to ensure they are prepared for postsecondary-level mathematics.

The Math Ready course focuses on the key readiness standards from the *Common Core State Standards for Mathematics* as well as the eight Standards for Mathematical Practice needed for students to be ready to undertake postsecondary academic or career preparation in non-STEM fields or majors. The course addresses content standards taught throughout high school, including content from Algebra I, Geometry, and Algebra II that are essential for college and careers.

The SREB Math Ready Course consists of seven mandatory modules (or units): algebraic expressions, equations, measurement and proportional reasoning, linear functions, linear systems of equations, quadratic functions, exponential functions, and an optional module on summarizing and interpreting statistical data. While this course covers the basics in math practices and reviews the procedural steps needed to be successful in math, it is designed to be taught in a new, engaging way based heavily on conceptual teaching and learning. Eight units comprise this course. They are described below.

Unit 1: Algebraic Expressions
The algebraic expressions unit was designed to solidify student understanding of expressions while providing the students with an opportunity to have success early in the course. The recurring theme integrated in this unit focuses on engaging students using and expanding the concepts found within purposefully chosen activities. Through guided lessons, students will manipulate, create and analyze algebraic expressions, and look at the idea of whether different sets of numbers are closed under certain operations.

Unit 2: Equations
The equations unit calls for students to construct and evaluate problems that involve one or two steps while seeking understanding of how and why equations and inequalities are used in their daily lives. Students also use the structure of word problems and equations to rewrite and solve equations in different forms revealing different relationships.

Mathematics | High School—SREB Math Ready Course

(continued)

Unit 3: Measurement and Proportional Reasoning
This unit deals with unit conversions, using proportions for scaling, and area and volume. The unit requires higher-order thinking and number sense in order to get to the true intent of the standards covered. It is useful in helping students make connections with math and science or other subjects.

Unit 4: Linear Functions
This unit takes students back to the foundation of all high school mathematics—an in-depth study of linear functions. Along with allowing students to differentiate between relations that are functions and those that are not, the unit helps students specifically examine characteristics of linear functions. By looking closely at linear functions in multiple forms, students are expected to graph and write equations, as well as interpret their meaning in context of the slope and y-intercept. Students conclude with a project allowing them to collect their own data and write a line of best fit from that data.

Unit 5: Linear Systems of Equations
The systems unit deals with solving systems of linear equations. This involves helping students classify solutions (one, none or infinitely many), as well as set up and solve problems using systems of equations. Students also choose the best way to solve a system of equations and explain their solutions.

Unit 6: Quadratic Functions
This unit is an expansive look at quadratic functions: their graphs, tables and algebraic functions. It stresses multiple approaches to graphing, solving and understanding quadratics, as students explore, make conjectures and draw conclusions in group-work settings. In this unit, students explore and learn from multiple applications of quadratics. The unit assumes students have seen quadratics before but may not have a concrete, transferrable understanding of quadratic functions. The unit does not cover algebraic manipulations (multiplying and factoring), as these are addressed in previous units.

Unit 7: Exponential Functions
This unit develops students' fluency in exponential functions through varying real-life financial applications/inquiries.

Unit 8: Summarizing and Interpreting Statistical Data (*optional*)
In this unit, students further develop skills to read, analyze, and communicate (using words, tables, and graphs) relationships and patterns found in data sets of one or more variables. Students learn how to choose the appropriate statistical tools and measurements to assist in analysis, communicate results, and read and inter interpret graphs, measurements, and formulas which are crucial skills in a world overflowing with data. Students explore these concepts while modeling real contexts based on data they collect.

Mathematics | High School—SREB Math Ready Course

(continued)

School districts that are interested in offering this course should visit http://www.sreb.org/page/1684/math_ready.html to review and download course materials.

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
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Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

*Note: Since AP Course Descriptions are updated regularly, please visit AP Central® (**apcentral.collegeboard.org**) to determine whether a more recent Course Description is available.*

AP courses in Calculus consist of a full high school academic year of work, and are one-credit courses comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the exam for each course. The two courses and the two corresponding exams are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still teach the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB Exam, most of the year must be devoted to the topics in differential and integral calculus. These topics are the focus of the AP Exam questions.

Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics taught in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics.

A *Calculus AB* subscore is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics. Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Exam or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies.

The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide statements regarding their AP policies in their catalogs and on their websites.

Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

(continued)

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses that are the equivalent of four full years of high school mathematics before attempting calculus. These courses should include the study of algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies — including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study — it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals of AP Calculus AB and AP Calculus BC:

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation, and should be able to use derivatives to solve a variety of problems.

- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and should be able to use integrals to solve a variety of problems.

Mathematics | High School—Advanced Placement (AP) Calculus AB Course Calculus BC Course

(continued)

- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

AP Central® (apcentral.collegeboard.org)

Interested parties can find the following Web resources at AP Central:

- AP Course Descriptions, information about the AP Course Audit, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.
- A searchable Institutes and Workshops database, providing information about professional development events.
- The Course Home Pages (apcentral.collegeboard.org/coursehomepages), which contain articles, teaching tips, activities, lab ideas, and other course-specific content contributed by colleagues in the AP community.
- Moderated electronic discussion groups (EDGs) for each AP course, provided to facilitate the exchange of ideas and practices.

Additional Resources

Teacher's Guides and Course Descriptions may be downloaded free of charge from AP Central; printed copies may be purchased through the College Board Store (store.collegeboard.org).

Advanced Placement (AP) Calculus AB
Advanced Placement (AP) Calculus BC

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Mathematics | High School—Advanced Placement (AP) Statistics Course

*Note: Since AP Course Descriptions are updated regularly, please visit AP Central® (**apcentral.collegeboard.org**) to determine whether a more recent Course Description is available.*

The AP statistics course, a one-credit course, introduces students to the major concepts and tools for collecting, analyzing and drawing conclusions from data. Students are exposed to four broad conceptual themes:

1. Exploring Data: Describing patterns and departures from patterns
2. Sampling and Experimentation: Planning and conducting a study
3. Anticipating Patterns: Exploring random phenomena using probability and simulation
4. Statistical Inference: Estimating population parameters and testing hypotheses

Students who successfully complete the course and exam may receive credit, advanced placement or both for a one-semester introductory college statistics course. This does not necessarily imply that the high school course should be one semester long. Each high school needs to determine the length of its AP Statistics course to best serve the needs of its students. The four themes are described below.

I. **Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns**.
In examining distributions of data, students should be able to detect important characteristics, such as shape, location, variability and unusual values. From careful observations of patterns in data, students can generate conjectures about relationships among variables. The notion of how one variable may be associated with another permeates almost all of statistics, from simple comparisons of proportions through linear regression. The difference between association and causation must accompany this conceptual development throughout.

II. Data must be collected according to a well-developed plan if valid information is to be obtained.
If data are to be collected to provide an answer to a question of interest, a careful plan must be developed. Both the type of analysis that is appropriate and the nature of conclusions that can be drawn from that analysis depend in a critical way on how the data was collected. Collecting data in a reasonable way, through either sampling or experimentation, is an essential step in the data analysis process.

Mathematics | High School—Advanced Placement (AP) Statistics Course (continued)

III. Probability is the tool used for anticipating what the distribution of data should look like under a given model.

Random phenomena are not haphazard: they display an order that emerges only in the long run and is described by a distribution. The mathematical description of variation is central to statistics. The probability required for statistical inference is not primarily axiomatic or combinatorial but is oriented toward using probability distributions to describe data.

IV. Statistical inference guides the selection of appropriate models*.*

Models and data interact in statistical work: models are used to draw conclusions from data, while the data are allowed to criticize and even falsify the model through inferential and diagnostic methods. Inference from data can be thought of as the process of selecting a reasonable model, including a statement in probability language, of how confident one can be about the selection.

AP Central® (apcentral.collegeboard.org)

Interested parties can find the following Web resources at AP Central:

- AP Course Descriptions, information about the AP Course Audit, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.
- A searchable Institutes and Workshops database, providing information about professional development events.
- The Course Home Pages (apcentral.collegeboard.org/coursehomepages), which contain articles, teaching tips, activities, lab ideas, and other course-specific content contributed by colleagues in the AP community.
- Moderated electronic discussion groups (EDGs) for each AP course, provided to facilitate the exchange of ideas and practices.

Additional Resources

Teacher's Guides and Course Descriptions may be downloaded free of charge from AP Central; printed copies may be purchased through the College Board Store (store.collegeboard.org).

Advanced Placement (AP) Statistics

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

Compensatory Mathematics Course Description

COMPENSATORY MATHEMATICS COURSE DESCRIPTION

Students in need of instructional support, intervention or remediation may be enrolled in a Compensatory Mathematics course under the following stipulations: The Compensatory mathematics course:

5. must be taken in concert with a credit-bearing course at the same grade level;
6. includes content supportive of the accompanying credit-bearing course;
7. should make every attempt to incorporate the Standards for Mathematical Practice; and
8. may be taken as an elective, but **will not** satisfy the number of mathematics Carnegie units required for graduation.

Standards for Mathematical Practice

9. Make sense of problems and persevere in solving them.
10. Reason abstractly and quantitatively.
11. Construct viable arguments and critique the reasoning of others.
12. Model with mathematics.
13. Use appropriate tools strategically.
14. Attend to precision.
15. Look for and make use of structure.
16. Look for and express regularity in repeated reasoning.

PARCC Assessment Guidance

PARCC
Partnership for Assessment of Readiness for College and Careers

PARCC Calculator Policy

Approved 7/23/12

Grades 3-5 Calculator Policy

- PARCC mathematics assessments for Grades 3 – 5 will not allow for calculator usage.

Grades 6-8 Calculator Policy

- PARCC mathematics assessments for Grades 6-7 will allow for an online four function calculator with square root.
- PARCC mathematics assessments for Grade 8 will allow for an online scientific calculator.
- PARCC mathematics assessments are to be divided into calculator and non-calculator sessions, provided that the other sessions of the assessment are locked.
- The same calculator with maximum functionality is to be used for all items on calculator sessions.

High School Calculator Policy

- PARCC mathematics assessments for High School will allow for an online calculator with functionalities similar to that of a TI-84 graphing calculator.
- PARCC mathematics assessments are to be divided into calculator and non-calculator sessions, provided that the other sessions of the assessment are locked.
- The same calculator with maximum functionality is to be used for all items on calculator sessions.

PARCC
Partnership for Assessment of Readiness for College and Careers

Assessment Reference Sheet

Approved 8/9/12

Grade 5

1 mile = 5,280 feet 1 mile = 1,760 yards	1 pound = 16 ounces 1 ton = 2,000 pounds	1 cup = 8 fluid ounces 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts 1 liter = 1000 cubic centimeters

Right Rectangular Prism	$V = Bh$ or $V = lwh$

Grade 6

1 inch = 2.54 centimeters 1 meter = 39.37 inches 1 mile = 5,280 feet 1 mile = 1,760 yards 1 mile = 1.609 kilometers	1 kilometer = 0.62 mile 1 pound = 16 ounces 1 pound = 0.454 kilograms 1 kilogram = 2.2 pounds 1 ton = 2,000 pounds	1 cup = 8 fluid ounces 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts 1 gallon = 3.785 liters 1 liter = 0.264 gallons 1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Right Rectangular Prism	$V = Bh$ or $V = lwh$

PARCC Reference Sheet
(continued)

Grade 7

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Grade 8

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pythagorean Theorem	$a^2 + b^2 = c^2$

PARCC Reference Sheet
(continued)

High School

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5,280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1,760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2,000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	$1 \; radian = \frac{180}{\pi} \; degree$
Degrees	$1 \; degree = \frac{\pi}{180} radian$
Exponential Growth/Decay	$A = A_0 \, e^{k(t-t_0)} + B_0$

Additional Resources

PARCC Model Content Frameworks
http://www.parcconline.org/parcc-model-content-frameworks
PARCC developed the Model Content Frameworks to help (1) inform development of item specifications and blueprints for the PARCC assessments, and (2) support implementation of the Common Core State Standards.

PARCC Calculator and Reference Sheet Policies
http://www.parcconline.org/assessment-administration-guidance
The PARCC reference sheets for grades 3 – High School have been developed based on the intent of the Common Core State Standards for Mathematics. One will notice that the names of the measurement formulas provided on the reference sheet only include the name of the figure or object in which the measurement formula pertains to. The intent of the CCSSM at grades 5 – 8 is to know and apply the measurement formulas. In order for students to be able to choose the correct formula, they will need to know the formula.

PARCC Sample Assessment Items
http://www.parcconline.org/samples/item-task-prototypes
The primary purpose of the PARCC sample assessment items is to provide information about the assessment system and support educators as they transition to the CCSSM and the PARCC assessments. The samples presented are designed to shed light on important elements of the CCSSM and to show how critical content in the standards may appear in PARCC's next-generation, technology-based assessments.

PARCC Performance Level Descriptors (PLDs)
http://www.parcconline.org/math-plds
The PARCC PLDs articulate the knowledge, skills, and practices that students performing at a given level should be able to demonstrate in each content area at each grade level. The ***grade- and subject-specific PLDs*** are intended to serve several purposes, including the following: (1) communicate expectations to educators about what types of performances will be necessary at the high school-level for students to demonstrate that they are college- and career-ready (CCR) or making adequate progress to become CCR; (2) communicate expectations to educators about what types of performance will be necessary in grades 3-8 for students to demonstrate that they are academically prepared to engage successfully in further studies in each content area; (3) provide information to local educators for use in developing curricular and instructional materials; (4) serve as the basis for PARCC standard setting in summer 2015; and (5) inform item and rubric development for the PARCC assessments.

Additional Resources (continued)

PARCC Assessment Blueprints/Evidence Tables
http://www.parcconline.org/assessment-blueprints-test-specs
Blueprints are a series of documents that together describe the content and structure of an assessment. These documents define the total number of tasks and/or items for any given assessment component, the standards measured, the item types, and the point values for each. *Evidence statement tables and evidence statements* describe the knowledge and skills that an assessment item or a task elicits from students. These are aligned directly to the Common Core State Standards, and highlight their advances especially around the coherent nature of the standards.

Glossary

Glossary

Note: The words that are defined here pertain to courses derived from the *Common Core State Standards for Mathematics*.

- **Addition and subtraction within 5, 10, 20, 100, or 1000**. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

- **Additive inverses**. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

- **Associative property of addition**. See Table 3 in this Glossary.

- **Associative property of multiplication.** See Table 3 in this Glossary.

- **Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

- **Box plot.** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

- **Commutative property**. See Table 3 in this Glossary.

- **Complex fraction**. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

- **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

- **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

- **Congruent**. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

- **Counting on**. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Glossary (continued)

- **Dot plot.** *See:* line plot.
- **Dilation**. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.
- **Expanded form**. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.
- **Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.
- **First quartile**. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.
- **Fraction**. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.
- **Identity property of 0**. See Table 3 in this Glossary.
- **Independently combined probability models**. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
- **Integer**. A number expressible in the form *a* or –*a* for some whole number *a*.
- **Interquartile Range**. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.
- **Line plot**. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]
- **Mean**. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Glossary (continued)

- **Mean absolute deviation**. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.
- **Median**. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.
- **Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.
- **Multiplication and division within 100**. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.
- **Multiplicative inverses**. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.
- **Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.
- **Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.
- **Probability distribution.** The set of possible values of a random variable with a probability assigned to each.
- **Properties of operations**. See Table 3 in this Glossary.
- **Properties of equality**. See Table 4 in this Glossary.
- **Properties of inequality**. See Table 5 in this Glossary.
- **Properties of operations**. See Table 3 in this Glossary.

Glossary (continued)

- **Probability**. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

- **Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

- **Random variable.** An assignment of a numerical value to each outcome in a sample space.

- **Rational expression.** A quotient of two polynomials with a non-zero denominator.

- **Rational number**. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

- **Rectilinear figure.** A polygon all angles of which are right angles.

- **Rigid motion.** A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

- **Repeating decimal**. The decimal form of a rational number. *See also:* terminating decimal.

- **Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered.

- **Scatter plot**. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

- **Similarity transformation**. A rigid motion followed by a dilation.

- **Tape diagram**. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Glossary (continued)

- **Terminating decimal.** A decimal is called terminating if its repeating digit is 0.
- **Third quartile**. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.
- **Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.
- **Uniform probability model**. A probability model which assigns equal probability to all outcomes. *See also:* probability model.
- **Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.
- **Visual fraction model.** A tape diagram, number line diagram, or area model.
- **Whole numbers**. The numbers 0, 1, 2, 3, ….

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

[4] To be more precise, this defines the *arithmetic mean*.

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Tables

Tables 1-5

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp.32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Source Code: *Miss. Code Ann.* § 37-1-3 (Adopted 02/2014)