Common Core Essential Elements for Mathematics

From the State Members of the

Dynamic Learning Maps
Alternate Assessment Consortium
and
Edvantia, Inc.

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INTRODUCTION

The Common Core Essential Elements (EEs) are linked to the Common Core State Standards (CCSS) for Mathematics. A group of general educators, special educators, and content specialists from member states in the Dynamic Learning Maps (DLM) Consortium gathered to determine the essence of the CCSS. The stakeholder group members were selected by their states to participate in this work. State education agency (SEA) representatives and SEA-selected teachers collaborated to develop the EEs.

This document provides a high-level view of the relationship between the CCSS and the links to performance for students with significant cognitive disabilities. It is intended to provide a beginning structure for the design of a summative alternate assessment. The document is not intended as a stand-alone guide to instruction, nor is it intended to contain all the steps in a complete learning progression or detailed curriculum. The DLM and associated professional development will provide greater detail than described in this document.

Beginning with the Mathematics CCSS, stakeholders defined links to illuminate the precursors for the essential content and skills contained in the grade level CCSS clusters and indicators. These EEs are not intended as a redefinition of the standards. Rather, they are intended to describe challenging expectations for students with significant cognitive disabilities in relation to the CCSS. The EEs clarify the bridge between grade level achievement expectations for students with significant cognitive disabilities who participate in alternate assessments and the CCSS.

Neither are the EEs intended to prescribe the beginning or end of instruction on the content and skills they represent; rather, they indicate the grade level at which initial mastery would be the target to be assessed. Students should begin instruction in content and skills at the earliest point possible and continue instruction until mastery is attained.

The stakeholder group also developed instructional achievement level descriptors (IALDs) for each of the EEs for four performance levels: I, II, III, and IV. For each IALD, the stakeholder group developed examples to illustrate how students might demonstrate achievement of the performance level across the broad range of students with significant cognitive disabilities. Both the IALDs and accompanying examples are available in a companion document available from the DLM Consortium.

Finally, the stakeholder group developed alternate assessment achievement descriptors for each grade level -- from third grade through high school -- where summative assessments might be required. The alternate assessment achievement descriptors will provide a bridge between the EEs and a summative alternate assessment aligned to them. The descriptors are intended to provide one element to guide development of the test blueprint, development of items and tasks that measure the full range of achievement, and the setting of cut scores during standard setting for the assessment. The focus of an alternate assessment in a standards-based system is based on the achievement that aligns with EEs linked to grade level content.

Together, the system of standards and descriptors is designed to allow students with significant cognitive disabilities to progress toward the achievement of state standards linked to grade level expectations. The relationship of standards and assessment to teaching and learning are depicted for use by teachers, assessment designers, and users of alternate assessment results.

NCLB GUIDANCE

The stakeholder group's work was guided by the U. S. Department of Education's Peer Review Guidance (*Standards and Assessments Peer Review Guidance: Information and Examples for Meeting Requirements of the No Child Left Behind Act of 2001 [NCLB]),* which requires that alternate academic achievement standards align with the alternate assessment. They must

- include knowledge and skills that link to grade level expectations,
- promote access to the general curriculum, and
- reflect professional judgment of the highest learning standards possible for the group of students with the most significant cognitive disabilities.

Although the grade-level content may be reduced in complexity or adjusted to reflect prerequisite skills, the link to grade-level standards must be clear. The Peer Review Guidance notes that the concept of alternate achievement standards related to grade level may be ambiguous. According to the Guidance, the descriptors

- should be defined in a way that supports individual growth because of their linkage to different content across grades;
- are not likely to show the same clearly defined advances in cognitive complexity as the general education standards when examined across grade levels;
- should rely on the judgment of experienced special educators and administrators, higher education representatives, and parents of students with disabilities as they define alternate achievement standards; and
- should provide an appropriate challenge for students with the most significant cognitive disabilities as they move through their schooling.

The Guidance requires links to grade-level standards. The EEs were developed by DLM consortium states to differentiate knowledge and skills by grade level. This differentiation is intended to clarify the link between the grade-level EEs and the grade-level CCSS and to show a forward progression across grades. The progression of content and skills across years of instruction reflect the changing priorities for instruction and learning as students move from grade to grade. The differences from grade level to grade level are often subtle and progression is sometimes

more horizontal than vertical. For example, the grade-to-grade level differences may consist of added skills that are not of obvious increasing rigor compared to the differences found in the CCSS across grade levels. To the degree possible, skills escalate in complexity or rigor at Levels III and IV across the grades, with clear links to the shifting emphasis at each grade level in the CCSS.

ACCESS TO INSTRUCTION AND ASSESSMENT

The EEs and Achievement Descriptors developed by the DLM consortium states are intended to create the maximum possible access to the CCSS for students with significant cognitive disabilities. The way in which information is presented for instruction and assessment and the manner in which students demonstrate achievement is in no way intended to be limited by statements of EEs or Achievement Descriptors. To that end, modes of communication, both for presentation or response, are not stated in either the EEs or Achievement Descriptors unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, graphic organizers similar to those used by students without disabilities provide useful access to content and are adequate to maximize opportunities to learn and demonstrate achievement. Other students require a range of assistive technologies to access content and demonstrate achievement. For some students, AAC devices and accommodations for hearing and visual impairments will be needed. As with other physical disabilities, students with visual impairments may perform some expectations using modified items, presentations, or response formats. A few items may not lend themselves to such modifications. Decisions about the appropriate modifications for visual impairments are accounted for in the design of the assessments.

The access challenge for some is compounded by the presence of multiple disabilities. All of these needs, as well as the impact of levels of alertness due to medication and other physical disabilities which may affect opportunities to respond appropriately, need to be considered.

Most presentation and response access conditions do not constitute accommodations as they are understood for students who take the general assessment. Methods of presentation that do not violate the intended construct by aiding or directing the students' response allow the student to perceive what knowledge or skill is expected. Aids to responding that do not constitute a violation of the intended construct allow the student to demonstrate the expected knowledge and skills. Examples of acceptable access technologies include the following:

- communication devices that compensate for a students' physical inability to produce independent speech.

- devices that compensate for a students' physical inability to manipulate objects or materials, point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

- tools that maximize a students' ability to acquire knowledge and skills and to demonstrate the products of their learning.

ACCESSING THE GENERAL CURRICULUM

Technology is also of particular importance to students with significant cognitive disabilities to access the general curriculum and achieve the EEs. Although educators have traditionally viewed technology as hardware and software, assistive technology tenets provide a broader view of the applications of low, medium, and high levels of technology use. Assistive technology tools can be vital to a student in acquiring and demonstrating learning unimpeded by the barriers that the disability presents.

Many students with significant cognitive disabilities have difficulty with or cannot use speech to communicate and/or are supported by the use of communication symbols (e.g., communication boards, speech generating devices, voice output communication devices) and supports to augment their speech and other means of communication. Students who require symbols and other AAC supports require frequent modeling in the use of those symbols to interact and respond during instruction. Students who use symbols and other communication supports need as much modeling as children who use speech to communicate. Modeling in this way is not viewed as a means of prompting, guidance, or support, just as having a teacher talk serves those purposes for a student who communicates using speech.

When modeling the use of symbols and other communication supports, teachers use the symbols and supports themselves, hand them to students without communication impairments to use, and involve the students who need to use them every day. Each of these steps can play an important role in validating the use of symbols and communication supports and demonstrating multiple levels of expertise in their use.

GUIDANCE AND SUPPORT

The authors of the CCSS use the words, "prompting and support" at the earliest grade levels to indicate when students were not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade, encourage, or remind someone to do or say something" (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to communicate clearly that teacher assistance is permitted during instruction of the EEs, and is not limited to structured prompting

procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include

- getting the student started (e.g., "Tell me what to do first"),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write dog but is unsure how, the teacher might say, "See if you can write the first letter in the word, /d/og."),
- narrowing the field of choices as a student provides an inaccurate response,
- using structured technologies such as task specific word banks, or
- providing the structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above apply to instruction. Alternate assessments measure the degree to which students with significant cognitive disabilities have mastered the EEs. During any assessment, accommodation(s) allowed on the assessment must have been used and practiced during instruction; however, some accommodations that are permissible during instruction would compromise the integrity of the assessments, thereby yielding invalid and unreliable results and cannot be used for assessment purposes. Some guidance and support strategies may not be allowed for assessment purposes when variance in teacher assistance, cues, and prompts could compromise judgments about mastery of the EEs and comparability of administration.

RELATIONSHIP TO THE DYNAMIC LEARNING MAPS ASSESSMENT

The EEs and Achievement Descriptors developed by the DLM consortium states and their stakeholder representatives provide teachers with information about the level of knowledge and skills expected of their students Assessment Achievement Level Descriptors (AALDs) will emerge as drafts. The AALDs are content and grade specific, but summarize across the EEs the key performance differences across levels of achievement and across grade levels. While draft AALDs will be used in the initial stages of standard setting to help guide that process, final AALDs will emerge from the standard setting process. Standard setting will take into account the overall degree of accuracy with which a student would need to perform in order to achieve at a particular level. Just as on a general education assessment, no individual student will be expected to perform proficiently on every EE in order to be considered Level III. (See Levels of Performance described below.)

For purposes of the DLM assessments under development, the achievement descriptors provide a useful link between the EEs and the DLM assessments. The descriptors, along with DLM developed from the CCSS, provide guidance to the development of the alternate

assessment so that a full range of performance is measured and the setting of score ranges within each level rests on a defined frame of reference. The grade level EEs and alternate achievement standards

- standardize meaning for the content and skill expectations,
- create consistency in expected performance,
- emphasize skill similarities for all students participating in the alternate assessments,
- accommodate diverse disabilities, and
- ground alternate assessments in a consistent set of expectations.

Achievement descriptors are used to categorize and explain student performance both in the course of instruction and on the alternate assessment.

SYSTEM ALIGNMENT

The EEs are intended to contribute to a fully aligned system of standards, curriculum, teaching, learning, technology, and assessment that optimize equity of opportunity for all students in each classroom, school, and local education agency to access and learn the standards. To the degree possible, the grade level EEs are vertically aligned and linked to the grade level CCSS.

The linkages provided by the EEs to the CCSS are intended to increase access to the general curriculum for all students with disabilities. Just as the EEs are designed to define achievement in academic content areas linked to the CCSS, the EEs reframe the expectations for foundational skills in pre-academic and academic areas. Precursor/prerequisite and the unique enabling skills related to mathematics content is specified in the context of their roles as a foundation for students with significant cognitive disabilities to achieve skills related to academic content.

Levels of Performance

Within this document, each grade level EE is cross-referenced to one or several CCSS.

Four performance levels have been proposed for the DLM's alternate academic achievement standards: I, II, III, and IV. Mastery is considered to be demonstrated at Level III and Level IV and is identified as meeting the Level III level on an alternate assessment as specified in the NCLB. A general description of each of these levels is included below:

Level I - A student at this level attempts to perform tasks with support.

Level II - A student at this level demonstrates some content knowledge and skills from the EEs linked to grade level standards.

Level III - A student at this level demonstrates content knowledge and skills at a level aligned with the complexity of the EEs.

Level IV - A student at this level demonstrates content knowledge and skills at a higher level of complexity than those described for Level III. Typically, this complexity includes the routine use of symbol systems as applied to mathematics.

For each performance level, specific descriptions of content and skills are bulleted and examples of each level of performance are provided. The EEs are intended as a resource for developing individualized education plan (IEP) goals, benchmarks, and curricular materials in reading, language arts, and mathematics. Students may need goals and benchmarks in areas other than academic content domains (e.g., self-care/living skills, mobility). As always, IEPs address the individual needs of each student to make progress toward the standards.

DOCUMENT ORGANIZATION

Common Core Grade-Level Clusters are the Cluster titles and Grade-Level Indicators as they appear in the CCSS for Mathematics (Common Core State Standards Initiative, 2010).

Common Core Essential Elements (EEs) describe links to the CCSS for access by students with significant cognitive disabilities.

CCSS Grade-Level Clusters	Common Core Essential Elements
Represent and solve problems involving addition and subtraction. **1.OA.1**. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**EE1.OA.1.a.** Use language to describe putting together and taking apart, aspects of addition and subtraction.

Directions for Interpreting Essential Elements

Essential Elements (EEs). The EEs are statements that provide links for students with significant cognitive disabilities to the essential content and skills defined in the grade-level clusters of the CCSS. The EEs provide a bridge for students with significant cognitive disabilities to the CCSS. The EEs are not intended as a reinterpretation of the CCSS; rather, they were developed to create a bridge between the CCSS and challenging achievement expectations for students with significant cognitive disabilities. The order in which the EEs are listed is a direct reflection of the order in which the CCSS are listed. The order is not intended to convey a sequence for instruction; rather, it illustrates progress across years. In the tables, the left column contains the CCSS grade-level clusters and indicators and the right column contains the EE linked to them.. Each EE completes the phrase "Students will . . . ."

CCSS marked with an (+) are advanced standards and are not included in this document as it was determined by the stakeholder group that students of this population would not be accessing the curriculum at this advanced level and writing Essential Elements to this level would be unnecessary. Also, if it appears that a standard has been omitted in the high school grades, it is an advanced standard.

"Begins in grade __" is utilized when the expectations for students to begin to demonstrate mastery is found at a higher grade level. Planning for instruction should incorporate instruction related to the higher grade level EE and begin at the earliest possible point for each student. Students with significant cognitive disabilities may require several years of instruction prior to the point at which they may be expected to begin to demonstrate mastery of a concept.

COMMON CORE ESSENTIAL ELEMENTS FOR KINDERGARTEN

Kindergarten Mathematics Standards: Counting and Cardinality

CCSS Grade-Level Clusters	Common Core Essential Elements
Know number names and the count sequence. **K.CC.1.** Count to 100 by ones and by tens.	**EEK.CC.1.** Starting with one, count to 10 by ones.
K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at one).	**EEK.CC.2.** N/A
K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	**EEK.CC.3.** N/A
Count to tell the number of objects. **K.CC.4.** Understand the relationship between numbers and quantities; connect counting to cardinality. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. Understand that each successive number name refers to a quantity that is one larger.	**EEK.CC.4.** Demonstrate one-to-one correspondence pairing each object with one and only one number and each name with only one object.

CCSS Grade-Level Clusters	Common Core Essential Elements
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	**EEK.CC.5.** Count out up to three objects from a larger set, pairing each object with one and only one number name to tell how many.
Compare numbers. **K.CC.6.** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.	**EEK.CC.6.** Identify whether the number of objects in one group is more or less than (when the quantities are clearly different) or equal to the number of objects in another group.
KK.CC.7. Compare two numbers between 1 and 10 presented as written numerals.	**EEK.CC.7.** N/A

Kindergarten Mathematics Standards: Operations and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. **K.OA.1.** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	**EEK.OA.1.** Represent addition as "putting together" or subtraction as "taking from" in everyday activities.
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	**EEK.OA.2.** N/A
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	**EEK.OA.3.** N/A
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	**EEK.OA.4.** N/A
K.OA.5. Fluently add and subtract within 5.	**EEK.OA.5.** N/A

[1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Kindergarten Mathematics Standards: Number and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Work with numbers 11-19 to gain foundations for place value. **K.NBT.1.** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**EEK.NBT.1.** N/A (See EEK.NBT.1.4 and EEK.NBT.1.6)

Kindergarten Mathematics Standards: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Describe and compare measurable attributes. **K.MD.1.** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.2.** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.** **K.MD.3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[2]	**EEK.MD.1-3.** Classify objects according to attributes (big/small, heavy/light).

[2] Limit category counts to be less than or equal to 10.

Kindergarten Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). **K.G.1.** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.	**EEK.G.1.** Identify words of proximity to describe the relative position.
K.G.2. Correctly name shapes regardless of their orientations or overall size. **K.G.3.** Identify shapes as two-dimensional (lying in a plane, "flat"; or three-dimensional, "solid").	**EEK.G.2-3.** Match two-dimensional shapes (circle, square, triangle).

COMMON CORE ESSENTIAL ELEMENTS FOR FIRST-GRADE

First Grade Mathematics Standards: Operations and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Represent and solve problems involving addition and subtraction. **1.OA.1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**EE1.OA.1.a.** Use language to describe putting together and taking apart, aspects of addition and subtraction.
	EE1.OA.1.b. Recognize two groups that have the same or equal quantity.
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**EE1.OA.2.** Use "putting together" to solve problems with two sets.
Understand and apply properties of operations and the relationship between addition and subtraction. **1.OA.3.** Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also*	**EE1.OA.3.** N/A

[3] Students need not use formal terms for these properties.

CCSS Grade-Level Clusters	Common Core Essential Elements
known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a 10, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)	
1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.*	**EE1.OA.4.** N/A (See EENBT.1.4 and EENBT.1.6)
Add and subtract within 20. **1.OA.5.** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**EE1.OA.5.a.** Use manipulatives or visual representations to indicate the number that results when adding one more.
	EE1.OA.5.b. Apply knowledge of "one less" to subtract one from the numbers.
1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**EE1.OA.6**. N/A
Work with addition and subtraction equations. **1.OA.7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	**EE1.OA.7.** N/A (See EE1.OA.1.b)

CCSS Grade-Level Clusters	Common Core Essential Elements
1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*	**EE1.OA.8.** N/A

First Grade Mathematics Standards: Number and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Extend the counting sequence. **1.NBT.1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**EE1.NBT.1.a.** Count by ones.
	EE1.NBT.1.b. Count as many as 10 objects and represent the quantity with the corresponding numeral.
Understand place value. **1.NBT.2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: ▪ 10 can be thought of as a bundle of ten ones — called a "ten." ▪ The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. ▪ The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	**EE1.NBT.2.** Create sets of 10.
1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**EE1.NBT.3.** Compare two groups of 10 or fewer items when the quantity of items in each group is similar.
Use place value understanding and properties of operations to	**EE1.NBT.4.** Compose numbers less than or equal to five in more

CCSS Grade-Level Clusters	Common Core Essential Elements
add and subtract. **1.NBT.4.** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	than one way.
1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	**EE1.NBT.5.** N/A (See EE1.OA.5.a and EE1.OA.5.b)
1.NBT.6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**EE1.NBT.6.** Decompose numbers less than or equal to five in more than one way.

First Grade Mathematics Standards: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Measure lengths indirectly and by iterating length units. **1.MD.1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **1.MD.2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	**EE1.MD.1-2.** Use appropriate vocabulary to describe the length of an object using the language of longer/shorter, taller/shorter.
Tell and write time. **1.MD.3.** Tell and write time in hours and half-hours using analog and digital clocks.	**EE1.MD.3.a.** Demonstrate an understanding of the terms "tomorrow, yesterday, and today."
	EE1.MD.3.b. Name a day of the week for tomorrow and yesterday.
	EE1.MD.3.c. Identify activities that come next, before, and after.
	EE1.MD.3.d. Demonstrate an understanding that telling time is the same every day.

CCSS Grade-Level Clusters	Common Core Essential Elements
Represent and interpret data. **1.MD.4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**EE1.MD.4.** Given a count of the total number of data points in two categories, determine whether there are more or less in each category.

First Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Reason with shapes and their attributes. **1.G.1.** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. **1.G.2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]	**EE1.G.1.** Identify common two-dimensional shapes: square, circle, triangle, and rectangle.
1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as *two of*, or *four of* the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**EE1.G.3.** Put together two pieces to make a shape that relates to the whole (i.e., two semicircles to make a circle, two squares to make a rectangle).

[4] Students do not need to learn formal names such as "right rectangular prism."

COMMON CORE ESSENTIAL ELEMENTS FOR SECOND GRADE

Second Grade Mathematics Standards: Operations and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Represent and solve problems involving addition and subtraction. **2.OA.1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	**EE2.OA.1.** Add and subtract to solve real world one-step story problems from 0-20 when the result is unknown.
Add and subtract within 20. **2.OA.2.** Fluently add and subtract within 20 using mental strategies.[5] By end of Grade 2, know from memory all sums of two one-digit numbers.	**EE2.OA.2.** N/A (See EE2.NBT.7)
Work with equal groups of objects to gain foundations for multiplication. **2.OA.3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even	**EE2.OA.3.** Equally distribute even numbers of objects between two groups.

[5] See standard 1.OA.6 for a list of mental strategies.

CCSS Grade-Level Clusters	Common Core Essential Elements
number as a sum of two equal addends.	
2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**EE2.OA.4.** Use addition to find the total number of objects arranged within equal groups up to a total of 10.

Second Grade Mathematics: Number and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand place value. **2.NBT.1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: ▪ 100 can be thought of as a bundle of ten tens — called a "hundred." ▪ The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	**EE2.NBT.1.** Represent numbers through 30 with sets of tens and ones with objects in columns or arrays.
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.	**EE2.NBT.2.a.** Count from 1 to 30 (count with meaning; cardinality).
	EE2.NBT.2.b. Name the next number in a sequence between 1 and 10.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**EE2.NBT.3.** Identify number symbols 1 to 30.
2.NBT.4. Compare two, three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**EE2.NBT.4.** Compare sets of objects and numbers using appropriate vocabulary (more, less, equal).
Use place value understanding and properties of operations to add and subtract.	**EE2.NBT.5.a.** Identify the meaning of the "+" sign (i.e., combine, plus, add), and the "=" sign (equal).

CCSS Grade-Level Clusters	Common Core Essential Elements
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	
	EE2.NBT.5.b. Using concrete examples, compose and decompose numbers up to 10 in more than one way.
2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. **2.NBT.7**. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	**EE2.NBT.6-7.** Use objects, representations, and numbers (0-20) to add and subtract.
2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. **2.NBT.9.** Explain why addition and subtraction strategies work, using place value and the properties of operations.[6]	**EE2.NBT.8-9.** N/A

[6] Explanations may be supported by drawings or objects.

Second Grade Mathematics: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Measure and estimate lengths in standard units. **2.MD.1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	**EE2.MD.1.** Measure the length of objects using non-standard units.
2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters. **2.MD.4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	**EE2.MD.3-4.** Order by length using non-standard units.
Relate addition and subtraction to length. **2.MD.5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	**EE2.MD.5.** Increase or decrease length by adding or subtracting unit(s).
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram.	**EE2.MD.6.** Use a number line to add one more unit of length.
Work with time and money.	**EE2.MD.7.** Indicate the digit that tells the hour on a digital clock.

CCSS Grade-Level Clusters	Common Core Essential Elements
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	
2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?	**EE2.MD.8.** Recognize that money has value.
Represent and interpret data. **2.MD.9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	**EE2.MD.9-10.** Create picture graphs from collected measurement data.

Second Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Reason with shapes and their attributes. **2.G.1.** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[7] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	**EE2.G.1.** Describe attributes of two-dimensional shapes.
2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	**EE2.G.2.** N/A
2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	**EE2.G.3.** N/A

[7] Sizes are compared directly or visually, not compared by measuring.

COMMON CORE ESSENTIAL ELEMENTS FOR THIRD GRADE

Third Grade Mathematics Standards: Operations and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Represent and solve problems involving multiplication and division. **3.OA.1.** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **3.OA.2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**EE3.OA.1-2.** Use repeated addition and equal groups to find the total number of objects to find the sum.
3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	**EE3.OA.3.** See EE3.OA.1. for repeated addition, a foundational skill for multiplication and division. (Multiplication begins in grade 4 and division begins in grade 5).
3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*	**EE3.OA.4.** Solve addition and subtraction problems when result is unknown with number 0-30.

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand properties of multiplication and the relationship between multiplication and division. **3.OA.5.** Apply properties of operations as strategies to multiply and divide.[8] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	**EE3.OA.5.** N/A (Multiplication begins at grade 4).
3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	**EE3.OA.6.** N/A (Division begins at grade 5).
Multiply and divide within 100. **3.OA.7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	**EE3.OA.7**. N/A (Multiplication begins grade 4 and division begins in grade 5).
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	**EE3.OA.8.** Add to solve real world one-step story problems from 0-30.

[8] Students need not use formal terms for these properties.

CCSS Grade-Level Clusters	Common Core Essential Elements
3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[9]	
3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**EE3.OA.9.** Identify arithmetic patterns.

[9] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.

Third Grade Mathematics Standards: Number and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Use place value understanding and properties of operations to perform multi-digit arithmetic.[10] **3.NBT.1**. Use place value understanding to round whole numbers to the nearest 10 or 100.	**EE3.NBT.1.** Identify the two 10s a number comes in between on a number line (numbers 0-30).
3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**EE3.NBT.2.** Identify place value to tens.
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**EE3.NBT.3**. Count by tens using money.

[10] A range of algorithms may be used.

Third Grade Mathematics Standards: Number and Operations--Fractions[11]

CCSS Grade-Level Clusters	Common Core Essential Elements
Develop understanding of fractions as numbers. **3.NF.1.** Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b.* **3.NF.2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram. ▪ Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. ▪ Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.NF.3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. ▪ Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	**EE3.NF.1-3.** Differentiate a fractional part from a whole.

[11] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

CCSS Grade-Level Clusters	Common Core Essential Elements
▪ Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. ▪ Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* ▪ Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

Third Grade Mathematics Standards: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. **3.MD.1**. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**EE3.MD.1.** Tell time to the hour on a digital clock.
3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[12] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[13]	**EE3.MD.2.** Identify standard units of measure for mass and liquid.
Represent and interpret data. **3.MD.3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**EE3.MD.3.** Use picture or bar graph data to answer questions about data.
3.MD.4. Generate measurement data by measuring lengths using	**EE3.MD.4.** Measure length of objects using standard tools, such

[12] Excludes compound units such as cm3 and finding the geometric volume of a container.
[13] Excludes multiplicative comparison problems (problems involving notions of "times as much".

CCSS Grade-Level Clusters	Common Core Essential Elements
rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	as rulers, yardsticks, and meter sticks.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition. **3.MD.5.** Recognize area as an attribute of plane figures and understand concepts of area measurement. ▪ A square with side length of 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. ▪ A plane figure, which can be covered without gaps or overlaps by n unit squares, is said to have an area of *n* square units. **3.MD.6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.7.** Relate area to the operations of multiplication and addition. ▪ Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. ▪ Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. ▪ Use tiling to show in a concrete case that the area of a	**EE3.MD.5-7.** N/A (Area begins at grade 6).

CCSS Grade-Level Clusters	Common Core Essential Elements
rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. ▪ Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. **3.MD.8.** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**EE3.MD.8.** N/A (Perimeter begins at grade 7).

Third Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Reason with shapes and their attributes. **3.G.1.** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	**EE3.G.1.** Recognize that shapes in different categories can share attributes.
3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**EE3.G.2.** Recognize that shapes can be partitioned into equal areas.

COMMON CORE ESSENTIAL ELEMENTS FOR FOURTH GRADE

Fourth Grade Mathematics Standards: Operations and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Use the four operations with whole numbers to solve problems. **4.OA.1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	**EE4.OA.1-2.** Demonstrate the connection between repeated addition and multiplication.
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**EE4.OA.3.** Solve one-step word problems using addition or subtraction.
Gain familiarity with factors and multiples. **4.OA.4.** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range	**EE4.OA.4.** Show one way to arrive at product.

CCSS Grade-Level Clusters	Common Core Essential Elements
1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	
Generate and analyze patterns. **4.OA.5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**EE4.OA.5.** Use repeating patterns to make predictions.

Fourth Grade Mathematics Standards: Numbers and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Generalize place value understanding for multi-digit whole numbers. **4.NBT.1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	**EE4.NBT.1.** Compare numbers to each other based on place value groups by composing and decomposing to 50.
4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**EE4.NBT.2.** Compare whole numbers (<, >, =).
4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.	**EE4.NBT.3.** Round one- and two-digit whole numbers from 0—50 to the nearest 10.
Use place value understanding and properties of operations to perform multi-digit arithmetic. **4.NBT.4.** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	**EE4.NBT 4.** Add and subtract double-digit whole numbers.

CCSS Grade-Level Clusters	Common Core Essential Elements
4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**EE4.NBT 5.** N/A (See EE. 4.OA.1.)
4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**EE4.NBT 6.** N/A

Fourth Grade Mathematics Standards: Number and Operations--Fractions[14]

CCSS Grade-Level Clusters	Common Core Essential Elements
Extend understanding of fraction equivalence and ordering. **4.NF.1.** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**EE4.NF.1-2.** Understand 2/4 = 1/2.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. **4.NF.3.** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. ▪ Understand addition and subtraction of fractions as joining	**EE4.NF.3.** Differentiate between whole, half, and fourth.

[14] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

and separating parts referring to the same whole. ▪ Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* ▪ Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. ▪ Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	
4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. ▪ Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* ▪ Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.).* ▪ Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does*	**EE4.NF.4.** N/A (See EE. 4.OA.1-2.)

your answer lie?	
Understand decimal notation for fractions, and compare decimal fractions. **4.NF.5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[15] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **4.NF.6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**EE4.NF.5.** N/A (Decimals begin at grade 7).

[15] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Fourth Grade Mathematics Standards: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. **4.MD.1.** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), . . .*	**EE4.MD.1.** Identify the smaller measurement units that divide a larger unit within a measurement system.
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**EE4.MD.2.a.** Tell time to the half hour using a digital or to the hour using an analog clock.
	EE4.MD.2.b. Select the appropriate measurement tool from two related options to solve problems.
	EE4.MD.2.c. Use standard measurement to compare lengths of objects.
	EE4.MD.2.d. Identify objects that have volume.
	EE4.MD.2.e. Identify coins (penny, nickel, dime, quarter) and

CCSS Grade-Level Clusters	Common Core Essential Elements
	their values.
4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**EE4.MD.3.** N/A (Area begins at 6th grade and perimeter begins at 7th grade).
Represent and interpret data. **4.MD.4.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	**EE4.MD.4.a.** Insert data into a preconstructed bar graph template.
	EE4.MD.4.b. Interpret data from a variety of graphs to answer questions.
Geometric measurement: understand concepts of angle and measure angles. **4.MD.5.** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: ▪ An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to	**EE4.MD.5.** Recognize angles in geometric shapes.

CCSS Grade-Level Clusters	Common Core Essential Elements
measure angles. ▪ An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**EE4.MD.6.** Identify angles as larger and smaller.
4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**EE4.MD.7.** N/A (See EE4.MD.5.)

Fourth Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Draw and identify lines and angles, and classify shapes by properties of their lines and angles. **4.G.1.** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**EE4.G.1.** Distinguish between parallel and intersecting lines.
4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	**EE4.G.2.** Distinguish between different attributes of shapes (lines, curves, angles).
4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	**EE4.G.3.** Recognize a line of symmetry in a simple shape.

COMMON CORE ESSENTIAL ELEMENTS FOR FIFTH GRADE

Fifth Grade Mathematics Standards: Operation and Algebraic Thinking

CCSS Grade-Level Clusters	Common Core Essential Elements
Write and interpret numerical expressions. **5.OA.1.** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.2.** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	**EE5.OA.1-2.** N/A

Analyze patterns and relationships. **5.OA.3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the	**EE5.OA.3.** Identify and extend numerical patterns.

CCSS Grade-Level Clusters	Common Core Essential Elements
two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	

Fifth Grade Mathematics Standards: Number and Operations in Base Ten

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand the place value system. **5.NBT.1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	**EE5.NBT.1.** Compare numbers to each other based on place value groups by composing and decomposing to 99.
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**EE5.NBT.2.** Recognize patterns in the number of zeros when multiplying a number by powers of 10.
5.NBT.3. Read, write, and compare decimals to 1000ths. ▪ Read and write decimals to 1000ths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). ▪ Compare two decimals to 1000ths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**EE5.NBT.3.** Round two-digit whole numbers to the nearest 10 from 0—90.
5.NBT.4. Use place value understanding to round decimals to any place.	**EE5.NBT.4.** Round money to a nearest dollar.
Perform operations with multi-digit whole numbers and with decimals to hundredths. **5.NBT.5.** Fluently multiply multi-digit whole numbers using the standard algorithm.	**EE5.NBT.5.** Multiply whole numbers up to 5 x 5.

CCSS Grade-Level Clusters	Common Core Essential Elements
5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7.** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**EE5.NBT.6-7.** Illustrate the concept of division using fair and equal shares.

Fifth Grade Mathematics Standards: Number and Operations--Fractions

CCSS Grade-Level Clusters	Common Core Essential Elements
Use equivalent fractions as a strategy to add and subtract fractions. **5.NF.1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd).*	**EE5.NF.1.** Differentiate between halves, fourths, and eighths.
5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	**EE5.NF.2.** Solve two-step word problems using addition and subtraction of whole numbers.
Apply and extend previous understandings of multiplication and division to multiply and divide fractions. **5.NF.3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4*	**EE5.NF.3.** N/A (See EE5.NF.1)

CCSS Grade-Level Clusters	Common Core Essential Elements
equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?	
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. ▪ Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* ▪ Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.5.** Interpret multiplication as scaling (resizing), by: ▪ Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. ▪ Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given	**EE5.NF.4-5.** N/A

CCSS Grade-Level Clusters	Common Core Essential Elements
number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[16] ▪ Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* ▪ Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*	**EE5.NF. 6-7**. N/A

[16] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

CCSS Grade-Level Clusters	Common Core Essential Elements
▪ Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	

Fifth Grade Mathematics Standards: Measurement and Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Convert like measurement units within a given measurement system. **5.MD.1.** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**EE5.MD.1.a.** Tell time using an analog or digital clock to the half or quarter hour.
	EE5.MD.1.b. Use customary units to measure weight and length of objects.
	EE5.MD.1.c. Indicate relative value of collections of coins.
Represent and interpret data. **5.MD.2.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**EE5.MD.2.a.** Represent and interpret data on a picture, line plot, or bar graph given a model and a graph to complete.
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.3.** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	**EE5.MD.3-5.** Determine volume of a cube by counting units of measure.

CCSS Grade-Level Clusters	Common Core Essential Elements
▪ A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. ▪ A solid figure, which can be packed without gaps or overlaps using *n* unit cubes, is said to have a volume of *n* cubic units. **5.MD.4.** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.5.** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. ▪ Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. ▪ Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. ▪ Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	

Fifth Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Graph points on the coordinate plane to solve real-world and mathematical problems. **5.G.1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.2.** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **5.G.3.** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. **5.G.4.** Classify two-dimensional figures in a hierarchy based on properties.	**EE5.G.1-5.** Sort two-dimensional figures and describe the common attributes such as angles, number of sides, corners (dimension), and color.

COMMON CORE ELEMENTS FOR SIXTH GRADE

Sixth Grade Mathematics Standards: Ratios and Proportional Relationships

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand ratio concepts and use ratio reasoning to solve problems. **6.RP.1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.2.** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[17] **6.RP.3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. ▪ Make tables of equivalent ratios relating quantities with	**EE6.RP.1.** Demonstrate a simple ratio relationship.

[17] Expectations for unit rates in this grade are limited to non-complex fractions.

CCSS Grade-Level Clusters	Common Core Essential Elements
whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. ▪ Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* ▪ Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. ▪ Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

Sixth Grade Mathematics Standards: The Number System

CCSS Grade-Level Clusters	Common Core Essential Elements
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. **6.NS.1.** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.*	**EE6.NS.1.** Compare the relationships between two unit fractions.
Compute fluently with multi-digit numbers and find common factors and multiples. **6.NS.2.** Fluently divide multi-digit numbers using the standard algorithm.	**EE6.NS.2**. Apply the concept of fair share and equal shares to divide.
6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**EE6.NS.3.** Solve two factor multiplication problems with products up to 50 using concrete objects and/or calculators.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two	**EE6.NS.4.** N/A

CCSS Grade-Level Clusters	Common Core Essential Elements
whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2). Apply and extend previous understandings of numbers to the system of rational numbers.*	
Apply and extend previous understandings of numbers to the system of rational numbers. **6.NS.5.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. ▪ Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. ▪ Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that	**EE6.NS.5-8.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

CCSS Grade-Level Clusters	Common Core Essential Elements
when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. ▪ Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.7.** Understand ordering and absolute value of rational numbers. ▪ Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* ▪ Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^{o}C > -7^{o}C$ to express the fact that $-3^{o}C$ is warmer than $-7^{o}C$.* ▪ Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* ▪ Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*	

CCSS Grade-Level Clusters	Common Core Essential Elements
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	

Sixth Grade Mathematics Standards: Expressions and Equations

CCSS Grade-Level Clusters	Common Core Essential Elements
Apply and extend previous understandings of arithmetic to algebraic expressions. **6.EE.1.** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.2.** Write, read, and evaluate expressions in which letters stand for numbers. ▪ Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* ▪ Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* ▪ Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*	**EE6.EE.1-2.** Identify equivalent number sentences.

CCSS Grade-Level Clusters	Common Core Essential Elements
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.4.** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities.*	**EE6.EE.3-4.** Demonstrate understanding of equivalent expressions.
Reason about and solve one-variable equations and inequalities. **6.EE.5.** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7.** Solve real-world and mathematical problems by writing	**EE6.EE.5-7.** Match an equation to a real-world problem in which variables are used to represent numbers.

CCSS Grade-Level Clusters	Common Core Essential Elements
and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **6.EE.8.** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
Represent and analyze quantitative relationships between dependent and independent variables. **6.EE.9.**Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	**EE6.EE.9.** N/A

Sixth Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Solve real-world and mathematical problems involving area, surface area, and volume. **6.G.1.** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real world and mathematical problems. **6.G.2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real world and mathematical problems.	**EE6.G.1-2.** Demonstrate area.
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	
6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**EE6.G.4.** Identify common three-dimensional shapes.

Sixth Grade Mathematics Standards: Statistics and Probability

CCSS Grade-Level Clusters	Common Core Essential Elements
Develop understanding of statistical variability. **6.SP.1.** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.2.** Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	**EE6.SP.1-2.** Display data on a graph or table that shows variability in the data.
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**EE6.SP.3.** N/A
Summarize and describe distributions. **6.SP.4.** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**EE6.SP.4.** N/A (See EE6.SP.1-2)
6.SP.5. Summarize numerical data sets in relation to their context, such as by: ▪ Reporting the number of observations. ▪ Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	**EE6.SP.5.** Summarize data distributions on a graph or table.

CCSS Grade-Level Clusters	Common Core Essential Elements
▪ Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. ▪ Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

COMMON CORE ESSENTIAL ELEMENTS FOR SEVENTH GRADE

Seventh Grade Mathematics Standards: Ratios and Proportional Relationships

CCSS Grade-Level Clusters	Common Core Essential Elements
Analyze proportional relationships and use them to solve real-world and mathematical problems. **7.RP.1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* **7.RP.2.** Recognize and represent proportional relationships between quantities. ▪ Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. ▪ Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. ▪ Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* ▪ Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit	**EE7.RP.1-3.** Use a ratio to model or describe a relationship.

CCSS Grade-Level Clusters	Common Core Essential Elements
rate. **7.RP.3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	

Seventh Grade Mathematics Standards: The Number System

CCSS Grade-Level Clusters	Common Core Essential Elements
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. **7.NS.1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. ▪ Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* ▪ Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. ▪ Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	**EE7.NS.1.** Add fractions with like denominators (halves, thirds, fourths, and tenths) so the solution is less than or equal to one.
Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.2.** Apply and extend previous understandings of	**EE7.NS.2.a.** Solve multiplication problems with products to 100.

CCSS Grade-Level Clusters	Common Core Essential Elements
multiplication and division and of fractions to multiply and divide rational numbers. ▪ Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	
▪ Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real-world contexts.	**EE7.NS.2.b.** Solve division problems with divisors up to five and also with a divisor of 10 without remainders.
▪ Apply properties of operations as strategies to multiply and divide rational numbers. ▪ Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**EE7.NS.2.c-d.** Compare fractions to fractions and decimals to decimals using rational numbers less than one.
7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers.[18]	**EE7.NS.3.** Demonstrate the value of various money amounts using decimals.

[18] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Seventh Grade Mathematics Standards: Expressions and Equations

CCSS Grade-Level Clusters	Common Core Essential Elements
Use properties of operations to generate equivalent expressions. **7.EE.1.** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	**EE7.EE.1-2.** Use the relationship within addition and/or multiplication to illustrate that two expressions are equivalent.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **7.EE.3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of*	**EE7.EE.3-4.** Use the concept of equality with models to solve one-step addition and subtraction equations.

CCSS Grade-Level Clusters	Common Core Essential Elements
a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. **7.EE.4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. ▪ Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* ▪ Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	

Seventh Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Draw construct, and describe geometrical figures and describe the relationships between them. **7.G.1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.2.** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	**EE7.G.1-2.** Draw or classify and recognize basic two-dimensional geometric shapes without a model (circle, triangle, rectangle/square).
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**EE7.G.3.** Match a two-dimensional shape with a three-dimensional shape that shares an attribute.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **7.G.4.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**EE7.G.4.** N/A
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**EE7.G.5.** Find the perimeter of a rectangle given the length and width.

CCSS Grade-Level Clusters	Common Core Essential Elements
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**EE7.G.6.** Find the area of a rectangle given the length and width using a model.

Seventh Grade Mathematics Standards: Statistics and Probability

CCSS Grade-Level Clusters	Common Core Essential Elements
Use random sampling to draw inferences about a population. **7.SP.1.** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.2.** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	**EE7.SP.1-2.** Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.
Draw informal comparative inferences about two populations. **7.SP.3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability*	**EE7.SP.3.** Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.

CCSS Grade-Level Clusters	Common Core Essential Elements
(mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. **7.SP.4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	
Investigate chance processes and develop, use, and evaluate probability models. **7.SP.5.** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.6.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly*	**EE7.SP.5-7.** Describe the probability of events occurring as possible or impossible.

CCSS Grade-Level Clusters	Common Core Essential Elements
200 times. **7.SP.7.** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. ▪ Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* ▪ Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	

COMMON CORE ESSENTIAL ELEMENTS FOR EIGHTH GRADE

Eighth Grade Mathematics Standards: The Number System

CCSS Grade-Level Clusters	Common Core Essential Elements
Know that there are numbers that are not rational, and approximate them by rational numbers. **8.NS.1.** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**EE8.NS.1.** Subtract fractions with like denominators (halves, thirds, fourths, and tenths) with minuends less than or equal to one.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations*	**EE8.NS.2.** Represent different forms and values of decimal numbers using fractions with numerators that are multiples of five and a denominator of 100.

Eighth Grade Mathematics Standards: Expressions and Equations

CCSS Grade-Level Clusters	Common Core Essential Elements
Expressions and Equations. Work with radicals and integer exponents. **8.EE.1.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.2.** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. **8.EE.3.** Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.* **8.EE.4.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).	**EE8.EE.1-4.** Compose and decompose numbers to three digits.

CCSS Grade-Level Clusters	Common Core Essential Elements
Interpret scientific notation that has been generated by technology.	
Understand the connections between proportional relationships, lines, and linear equations. **8.EE.5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **8.EE.6.** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	**EE8.EE.5-6.** Graph a simple ratio using the x and y axis points when given the ratio in standard form (2:1) and convert to 2/1.
Analyze and solve linear equations and pairs of simultaneous linear equations. **8.EE.7.** Solve linear equations in one variable. ▪ Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). ▪ Solve linear equations with rational number	**EE8.EE.7.** Solve algebraic expressions using simple addition and subtraction.

CCSS Grade-Level Clusters	Common Core Essential Elements
coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
8.EE.8. Analyze and solve pairs of simultaneous linear equations. ▪ Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. ▪ Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* ▪ Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	**EE8.EE.8.** N/A (See EE.8.EE.5-6)

Eighth Grade Mathematics Standards: Functions

CCSS Grade-Level Clusters	Common Core Essential Elements
Define, evaluate, and compare functions. **8.F.1.** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[19] **8.F.2.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **8.F.3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	**EE8.F.1-3.** Given a function table, identify the missing number.
Use functions to model relationships between quantities.	**EE8.F.4.** Determine the values or rule of a function using a graph or a table.

[19] Function notation is not required in Grade 8.

CCSS Grade-Level Clusters	Common Core Essential Elements
8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	
8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**EE8.F.5.** Describe how a graph represents a relationship between two quantities.

Eighth Grade Mathematics Standards: Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand congruence and similarity using physical models, transparencies, or geometry software. **8.G.1.** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. **8.G.2.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.3.** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	**EE8.G.1-3.** Identify similarity and congruence (same) in objects and shapes containing angles without translations.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**EE8.G.4.** Identify similar shapes with and without rotation.
8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created	**EE8.G.5.** Compare measures of angles to a right angle (greater than, less than, or equal to).

CCSS Grade-Level Clusters	Common Core Essential Elements
when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	
Understand and apply the Pythagorean Theorem. **8.G.6.** Explain a proof of the Pythagorean Theorem and its converse. **8.G.7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.G.8.** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**EE8.G.6-8.** N/A
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. **8.G.9.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**EE8.G.9.** Identify volume of common measures (cups, pints, quarts, gallons, etc.).

Eighth Grade Mathematics Standards: Statistics and Probability

CCSS Grade-Level Clusters	Common Core Essential Elements
Investigate patterns of association in bivariate data. **8.SP.1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.2.** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	**EE8.SP.1-3.** N/A
8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies	**EE8.SP.4.** Construct a graph or table from given categorical data and compare data categorized in the graph or table.

CCSS Grade-Level Clusters	Common Core Essential Elements
calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	

COMMON CORE ESSENTIAL ELEMENTS FOR HIGH SCHOOL

High School Mathematics Standards: Number and Quantity - The Real Number System

CCSS Grade-Level Clusters	Common Core Essential Elements
Extend the properties of exponents to rational exponents. **N-RN.1.** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**EEN-RN.1.** Solve division problems with remainders using concrete objects.
N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**EEN-RN.2.** N/A
Use properties of rational and irrational numbers. **N-RN.3**. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**EEN-RN.3**. N/A

High School Mathematics Standards: Number and Quantity - Quantities

CCSS Grade-Level Clusters	Common Core Essential Elements
Reason quantitatively and use units to solve problems. **N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**EEN-Q.1-3.** Express quantities to the appropriate precision of measurement.

High School Mathematics Standards: Number and Quantity - The Complex Number System

CCSS Grade-Level Clusters	Common Core Essential Elements
Perform arithmetic operations with complex numbers. **N-CN.1**. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.	**EEN-CN.1.** N/A
N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**EEN-CN.2.** Use the operations of addition, subtraction, and multiplication with decimals (decimal value x whole number) in real world situations using money as the standard units ($20, $10, $5, $1, $0.25, $0.10, $0.05, and $0.01).
Use complex numbers in polynomial identities and equations. **N-CN.7.** Solve quadratic equations with real coefficients that have complex solutions.	**EEN-CN.7.** N/A

High School Mathematics Standards: Algebra - Seeing Structure in Expressions

CCSS Grade-Level Clusters	Common Core Essential Elements
Interpret the structure of expressions. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context. ▪ Interpret parts of an expression, such as terms, factors, and coefficients. ▪ Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	**EEA-SSE.1.** Match an algebraic expression involving one operation to represent a given word expression with an illustration.
A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*	**EEA-SSE.2.** N/A
Write expressions in equivalent forms to solve problems. **A-SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the*	**EEA-SSE.3.** Solve simple one-step equations (multiplication and division) with a variable.

CCSS Grade-Level Clusters	Common Core Essential Elements
approximate equivalent monthly interest rate if the annual rate is 15%.	
A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	**EEA-SSE.4** Identify the missing part in any other equivalent ratio when given any ratio.

High School Mathematics Standards: Algebra - Arithmetic with Polynomials and Rational Expressions

CCSS Grade-Level Clusters	Common Core Essential Elements
Perform arithmetic operations on polynomials. **A-APR.1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**EEA-APR.1** N/A

High School Mathematics Standards: Algebra - Creating Equations

CCSS Grade-Level Clusters	Common Core Essential Elements
Create equations that describe numbers or relationships. **A-CED.1.** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	**EEA-CED.1.** Solve an algebraic expression using subtraction.
A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-CED.3.** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **A-CED.4.** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	**EEA-CED.2-4.** Solve one-step inequalities.

High School Mathematics Standards: Algebra - Reasoning with Equations and Inequalities

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand solving equations as a process of reasoning and explain the reasoning. **A-REI.1.** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A-REI.2.** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**EEA-REI.1-2.** N/A
Solve equations and inequalities in one variable. **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A-REI.4.** Solve quadratic equations in one variable. ▪ Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. ▪ Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives	**EEA-REI.3.** N/A (See EEA-ECED.1-2.)

CCSS Grade-Level Clusters	Common Core Essential Elements
complex solutions and write them as $a \pm bi$ for real numbers a and b.	
Solve systems of equations. **A-REI.5.** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**EEA-REI.5.** N/A
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **A-REI.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.	**EEA-REI.6-7.** N/A (See EEA-REI.10-12.)
Represent and solve equations and inequalities graphically. **A-REI.10.** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A-REI.11.** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial,	**EEA-REI.10.-12.** Determine the two pieces of information that are plotted on a graph of an equation with two variables that form a line when plotted.

CCSS Grade-Level Clusters	Common Core Essential Elements
rational, absolute value, exponential, and logarithmic functions. **A-REI.12.** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

High School Mathematics Standards: Functions - Interpreting Functions

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand the concept of a function and use function notation. **F-IF.1.** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **F-IF.2**. Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.3.** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	**EEF-IF.1-3.** Use the concept of function to solve problems.
Interpret functions that arise in applications in terms of the context. **F-IF.4**. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include*	**EEF-IF.4-6.** Interpret rate of change (e.g., higher/lower, faster/slower).

CCSS Grade-Level Clusters	Common Core Essential Elements
intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **F-IF.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F-IF.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	
Analyze functions using different representations. **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**EEF-IF.7.** N/A (See EEF-IF.1-3)

CCSS Grade-Level Clusters	Common Core Essential Elements
F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.	**EEF-IF.8.** N/A
F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**EEF-IF.9.** N/A

High School Mathematics Standards: Functions - Building Functions

CCSS Grade-Level Clusters	Common Core Essential Elements
Build a function that models a relationship between two quantities. **F-BF.1.** Write a function that describes a relationship between two quantities. ▪ Determine an explicit expression, a recursive process, or steps for calculation from a context. ▪ Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**EEF-BF.1.** Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.
F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	**EEF-BF.2.** Build an arithmetic sequence when provided a recursive rule with whole numbers.
Build new functions from existing functions. **F-BF.3**. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **F-BF.4**. Find inverse functions. Solve an equation of the form f(x)	**EEF-BF.3-4.** N/A

CCSS Grade-Level Clusters	Common Core Essential Elements
= c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1.*	

High School Mathematics Standards: Functions - Linear, Quadratic, and Exponential Models

CCSS Grade-Level Clusters	Common Core Essential Elements
Construct and compare linear, quadratic, and exponential models and solve problems. **F-LE.1.** Distinguish between situations that can be modeled with linear functions and with exponential functions. ▪ Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ▪ Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ▪ Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F-LE.2**. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **F-LE.3.** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **F-LE.4**. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	**EEF-LE.1.** Model a simple linear function such as y=mx to show functions grow by equal factors over equal intervals.

CCSS Grade-Level Clusters	Common Core Essential Elements
Interpret expressions for functions in terms of the situation they model. **F-LE.5.** Interpret the parameters in a linear or exponential function in terms of a context.	**EEF-LE.5.** N/A

High School Mathematics Standards: Functions - Trigonometric Functions

CCSS Grade-Level Clusters	Common Core Essential Elements
Extend the domain of trigonometric functions using the unit circle. **F-TF.1**. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2**. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**EEF-TF.1-2**. N/A
Model periodic phenomena with trigonometric functions. **F-TF.5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**EEF-TF.5.** N/A
Prove and apply trigonometric identities. **F-TF.8**. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	**EEF-TF.8**. N/A

High School Mathematics Standards: Geometry - Congruence

CCSS Grade-Level Clusters	Common Core Essential Elements
Experiment with transformations in the plane. **G.CO.1**. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	**EEG-CO.1.** Know the attributes of perpendicular lines, parallel lines, and line segments, angles, and circles.
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	**EEG-CO.2.** N/A
G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	**EEG-CO.3.** N/A
G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.5.** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**EEG-CO.4-5.** Identify rotations, reflections, and slides.

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand congruence in terms of rigid motions. **G-CO.6.** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **G-CO.7.** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G-CO.8**. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	**EEG-CO.6-8.** Identify corresponding congruent (the same) parts of shapes.

CCSS Grade-Level Clusters	Common Core Essential Elements
Prove geometric theorems **G-CO.9.** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G-CO.10.** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G-CO.11.** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	**EEG-CO.9-11.** N/A

CCSS Grade-Level Clusters	Common Core Essential Elements
Make geometric constructions. **G-CO.12.** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G-CO.13.** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**EEG-CO.12-13.** N/A

High School Mathematics Standards: Geometry - Similarity, Right Triangles, and Trigonometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand similarity in terms of similarity transformations. **G-SRT.1.** Verify experimentally the properties of dilations given by a center and a scale factor: ▪ A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. ▪ The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G-SRT.2.** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G-SRT.3.** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**EEG-SRT.1-3.** N/A (See EEG-CO.6-8.)
Prove theorems involving similarity. **G-SRT.4.** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G-SRT.5.** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	**EEG-SRT.4-5.** N/A

CCSS Grade-Level Clusters	Common Core Essential Elements
Define trigonometric ratios and solve problems involving right triangles. **G-SRT.6.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G-SRT.7.** Explain and use the relationship between the sine and cosine of complementary angles. **G-SRT.8.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**EEG-SRT.6-8.** N/A

High School Mathematics Standards: Geometry - Circles

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand and apply theorems about circles. **G-C.1.** Prove that all circles are similar. **G-C.2.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G-C.3.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**EEG-C.1-3.** N/A
Find arc lengths and areas of sectors of circles. **G-C.5**. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	**EEG-C.5.** N/A

High School Mathematics Standards: Geometry - Expressing Geometric Properties with Equations

CCSS Grade-Level Clusters	Common Core Essential Elements
Translate between the geometric description and the equation for a conic section. **G-GPE.1.** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**EEG-GPE.1.** N/A
G-GPE.2. Derive the equation of a parabola given a focus and directrix.	**EEG-GPE.2-4.** N/A
Use coordinates to prove simple geometric theorems algebraically. **G-GPE.4.** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	**EEG-GPE.4.** N/A (See EEG-GPE)
G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G-GPE.6.** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**EEG-GPE.5-6.** N/A (See EEG.CO.1)

CCSS Grade-Level Clusters	Common Core Essential Elements
G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	**EEG-GPE.7.** Find perimeter and area of squares and rectangles to solve real-world problems.

High School Mathematics Standards: Geometry - Geometric Measurement and Dimension

CCSS Grade-Level Clusters	Common Core Essential Elements
Explain volume formulas and use them to solve problems. **G-GMD.1.** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G-GMD.3.** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	**EEG-GMD.1-3.** Make a prediction based on knowledge of volume to identify volume of common containers (cups, pints, gallons, etc.).
Visualize relationships between two-dimensional and three-dimensional objects. **G-GMD.4**. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**EEG-GMD.4.** Distinguish between two-dimensional and three-dimensional objects to solve real-world problems.

High School Mathematics Standards: Geometry - Modeling with Geometry

CCSS Grade-Level Clusters	Common Core Essential Elements
Apply geometric concepts in modeling situations. **G-MG.1**. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **G-MG.2.** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **G-MG.3.** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	**EEG-MG.1-3**. Use properties of geometric shapes to describe real-life objects.

High School Mathematics Standards: Statistics and Probability - Interpreting Categorical and Quantitative Data

CCSS Grade-Level Clusters	Common Core Essential Elements
Summarize, represent, and interpret data on a single count or measurement variable. **S-ID.1.** Represent data with plots on the real number line (dot plots, histograms, and box plots). **S-ID.2.** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**EES-ID.1-2**. Given data, construct a simple graph (table, line, pie, bar, or picture) and answer questions about the data.
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**EES-ID.3.** Indicate general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**EES-ID.4.** Calculate the mean of a given data set (limit data points to less than five).

CCSS Grade-Level Clusters	Common Core Essential Elements
Summarize, represent, and interpret data on two categorical and quantitative variables. **S-ID.5.** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **S-ID.6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	**EES-ID.5.** N/A (See EEF-IF.1. and EEA-REI.6-7)
Interpret linear models. **S-ID.7.** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**EES-ID.7.** N/A (See EEF.IF.4-6)
S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. **S-ID.9.** Distinguish between correlation and causation.	**EES-ID.8-9.** N/A

High School Mathematics Standards: Statistics and Probability - Making Inferences and Justifying Conclusions

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand and evaluate random processes underlying statistical experiments. **S-IC.1.** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S-IC.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	**EES-IC.1-2.** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
Make inferences and justify conclusions from sample surveys, experiments, and observational studies. **S-IC.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S-IC.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S-IC.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	**EES-IC.3-6.** N/A (See EES-ID.1-2)

CCSS Grade-Level Clusters	Common Core Essential Elements
S-IC.6. Evaluate reports based on data.	

High School Mathematics Standards: Statistics and Probability - Conditional Probability and the Rules of Probability

CCSS Grade-Level Clusters	Common Core Essential Elements
Understand independence and conditional probability and use them to interpret data. **S-CP.1.** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S-CP.2.** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S-CP.3.** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **S-CP.4.** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that*	**EES-CP.1-4.** Identify when events are independent or dependent.

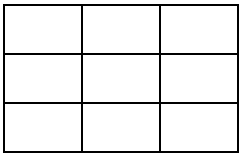
CCSS Grade-Level Clusters	Common Core Essential Elements
the student is in tenth grade. Do the same for other subjects and compare the results. **S-CP.5.** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	
Use the rules of probability to compute probabilities of compound events in a uniform probability model. **S-CP.6.** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **S-CP.7.** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	**EES-CP.6-7.** N/A (See EES-IC.1-2)

GLOSSARY AND EXAMPLES OF MATHEMATICS TERMS

Acute triangle. A triangle with all acute angles (acute means measuring less than 90°). See http://www.mathsisfun.com/definitions/acute-triangle.html

Angles. A shape formed by two lines or rays that diverge from a common point or vertex.

Area. The size of a region enclosed by the figure. Area is measured in square units (e.g., the area of this rectangle is six square units).



Associative property for addition. The sum of three or more numbers which are always the same when added together, no matter what order they are in. This is illustrated by $a + (b + c) = (a + b) + c$; $2 + (3 + 4) = (2 + 3) + 4$.

Associative property for multiplication. The product of three or more numbers which are always the same when multiplied together, regardless of their grouping. This is illustrated by $a(bc) = (ab)c$; $2(3\times4) = (2\times3)4$.

Attributes. For math purposes, "attributes" refer to characteristics of an object or geometric shape. These include qualities of shape, color, size, side, length, etc.

Base ten blocks. Blocks used to learn place value, addition, subtraction, multiplication, and division. Base ten blocks consist of cubes (ones place), rods (tens place), flats (hundreds place), and blocks (thousands place).

Categorical data. Types of data, which may be divided into groups such as race, sex, age group, and educational level when categorized into a small number of groups.

Commutative property of addition. The sum of numbers are always the same when added together, no matter if the order of the addends are changed. This is illustrated by $a + b = b + a$ $(2 + 1 = 1 + 2)$.

Commutative property of multiplication. The product of numbers are always the same when multiplied together, even if the order of factors are changed (i.e., if a and b are two real numbers, then $a \times b = b \times a$.)

Compose numbers. To combine parts/components to form a number (adding parts to obtain a number).

Congruent figures. Figures that have the same size and shape.

Congruent/congruence. The same.

Decompose numbers. The process of separating numbers into their components (to divide a number into smaller parts). *Example:* 456 can be decomposed as 456 = 400 + 50 + 6.

Denominator. The "bottom" number of a fraction; the number that represents the total number of parts into which one whole is divided (e.g., in 3/4, the 4 is the denominator and indicates that one whole is divided into 4 parts).

Dividend. The number that is being divided (e.g., In the problem, there are 550 pencils; each pack has 10 pencils; how many packs are there? 550 ÷ 10 = 55, *550* is the dividend because it tells how many pencils there are in all to be divided.).

Divisor. A number by which another number is divided (e.g., In the problem, there are 550 pencils; each pack has 10 pencils; how many packs are there? 550 ÷ 10 = 55, *10* is the divisor because it tells how many times 550 is to be divided.

Edge. The line segment where two faces of a solid figure meet (i.e., a cube has 12 edges).

ELA. English Language Arts

Equation. A mathematical sentence of equality between two expressions; equations have an equal sign (e.g., n + 50 = 75 or 75 = n + 50 means that n + 50 must have the same value as 75).

Equilateral triangle. A triangle with all three sides of equal length, corresponding to what could also be known as a "regular" triangle – an equilateral triangle is therefore a special case of an isosceles triangle having not just two but all three sides equal. An equilateral triangle also has three equal angles. See http://www.mathsisfun.com/definitions/equilateral-triangle.html

Expression. An operation between numbers that represents a single numeric quantity; expressions do not have an equal sign (e.g., *4r, x+2, y-1*).

Face. A plane surface of a three-dimensional figure.

Fact families. Sets of related math facts. For example:

> Addition fact family: 3 + 5 = 8; 8 - 3 = 5; 5 + 3 = 8; and 8 - 5 = 3
> Multiplication fact family: 5 x 4 = 20; 20 ÷ 5 = 4; 4 x 5=20; and 20 ÷ 4 = 5

Fair share. In division meaning splitting into equal parts or groups with nothing left over.

Frequency table. A table that lists items and uses tally marks to record and show the number of times they occur.

Functions. A special kind of relation where each x-value has one and only one y-value.

Function table. A table that lists pairs of numbers that show a function.

Inequality. A mathematical sentence in which the value of the expressions on either side of the relationship symbol are unequal; relation symbols used in inequalities include > (greater than) and < (less than) symbols (e.g., $7 > 3$, $x < y$).

Input/output table. A table that lists pairs of numbers that show a function.

Integers. Positive and negative whole numbers.

Interlocking cubes. Manipulatives that help students learn number and math concepts - cubes represent "units" and link in one direction. Interlocking cubes are used for patterning, grouping, sorting, counting, numbers, addition, subtraction, multiplication, division, and measurement.

Intersecting lines. Lines that cross.

Inverse operations. Opposite/reverse operations (e.g., subtraction is the inverse operation of addition, which is why 4 + 5 = 9 and 9 – 5 = 4; division is the inverse operation of multiplication, which is why 4 x 5 = 20 and 20 ÷ 5 = 4).

Linear equation. An equation that is made up of two expressions set equal to each other (e.g., $y = 2x + 5$) - A linear equation has only one or two variables and graph as a straight line. See http://www.eduplace.com/math/mathsteps/7/d/index.html

Line graph. A graphical representation using points connected by line segments to show how something changes over time.

Lines of symmetry. Any imaginary line along which a figure could be folded so that both halves match exactly.

Manipulatives. Objects that are used to explore mathematical ideas and solve mathematical problems (e.g., tools, models, blocks, tiles cubes, geoboards, colored rods, M&M's).

Mathematical structures.

Addition – compare-total unknown
Ex. If Anita has 10 sheets of paper and you have 10 more sheets than Anita. How many sheets do you have?

Addition – start unknown
Ex. Sam gave away 10 apples and has five apples left. How many apples did he start have before he gave 10 apples?

Addition join-part/part – whole
Ex. Jessie had 20 cakes and bought five more. How many does he have now?

Subtraction – classic take away
Ex. If Judy had $50 and spent $10, how much does she have left?

Subtraction – difference unknown
Ex. Sandi has 10 cats and 20 dogs. Which does she have more of, cats or dogs? How many more?

Subtraction – deficit missing amount
Ex. Sandy wants to collect 35 cards and she already has 15. How many more cards does she need?

Multiplication – repeated addition
Ex. James got paid $5 each day for five days. How much money did he have at the end of the five days?

Multiplication – array
Ex. Carlos wanted to cover his rectangular paper with one-inch tiles. If his paper is five inches long and four inches wide, how many tiles will it take to cover the paper?

Multiplication – fundamental counting principle
Ex. Julie packed four shirts and four jeans for her trip. How many outfits can she make?

Division – repeated subtraction
Ex. James pays $5 each day to ride the bus. How many days can he ride for $20?

Division – factor/area – side length
Ex. Tim wants to know the width of a rectangular surface covered in 20 one-inch tiles. He knows the length is five inches, but what is the width?

Division – partitive/fair share
Ex. Julie has 20 different outfits. She has five shirts – how many pair of jeans does she have to make 20 different outfits?

Mean. The "average" – To find the mean, add up all the numbers and then divide by the number of numbers.

Median. The "middle" value in the list of numbers - To find the median, your numbers have to be listed in numerical order, so you may have to rewrite your list.

Minuend. The number one is subtracting from (e.g., 9 in 9 – 2 = __).

Mode. The value that occurs most often - If no number is repeated, then there is no mode for the list. See http://www.purplemath.com/modules/meanmode.htm

Models. Pictorial or tactile aids used explore mathematical ideas and solve mathematical problems – Manipulatives can be used to model situations.

Non-numeric patterns. Using symbols, shapes, designs, and pictures to make patterns (e.g., □□ΔΔ◊◊□□ΔΔ◊◊).

Non-standard units of measure. Measurements that are neither metric nor English (e.g., number of footsteps used to measure distance or using a piece of yarn used to measure length).

Number line. A diagram that represents numbers as points on a line; a number line must have the arrows at the end.

Number sentence. An equation or inequality using numbers and symbols that is written horizontally (e.g., 5 < 7 or 5 +7+12).

Numerals. 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Numeric patterns. A pattern that uses skip counting, often starting with the number 1 or 2 – Counting by tens and twos may also be presented to students beginning with different numbers such as 7 or 23; this is more difficult for students but indicates a deeper understanding of skip counting (e.g., 7, 17, 27, 37, 47, . . . or 7, 9, 11, 13, 15, 17).

Numerical expression. A mathematical phrase that involves only numbers and one or more operational symbols.

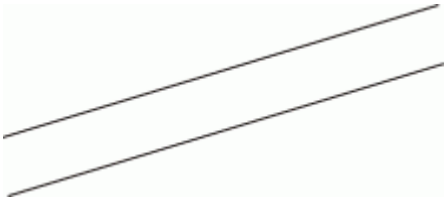
Obtuse triangle. A triangle that has one obtuse angle (obtuse means measuring more than 90°). See http://www.mathsisfun.com/definitions/obtuse-triangle.html

Operations. Addition, subtraction, multiplication, and division.

Ordered pair. In the ordered pair (1, 3), the first number is called the x-coordinate; the second number is called the y-coordinate; this ordered pair represents the coordinates of point A.

- The x-coordinate tells the distance right (positive) or left (negative).
- The y-coordinate tells the distance up (positive) or down (negative).

Parallel Lines. Lines that are the same distance apart and that never intersect – Lines that have the same slope are parallel.



Pattern. Patterns with a minimum of three terms

- using numbers by repeatedly adding or subtracting (i.e., 2, 4, 6, 8, 10, 12; 0, 3, 6, 9, 12, 15; or 50, 45, 40, 35, 30, 25).
- using objects, figures, colors, sound, etc. - a repeated pattern needs to be at least six terms.

  Extend a pattern - When a student is asked to continue a pattern, the pattern is presented, and the student is asked, "What comes next?" before a student can extend or describe a pattern, the given pattern must be comprised of a minimum of three terms so that the student can see the regularities of the situation and extend or describe the pattern based on those regularities.

Percent. A way of expressing a fraction as "out of 100" (e.g., 50% means 50 out of 100 or 50/100).

Perpendicular lines. Lines that intersect, forming right angles.

Polygon. A closed plane figure made by line segments.

Prediction. A guess based on available information.

Quadrilateral. A four-sided polygon.

Rational numbers. Any number that can be expressed as a/b (b≠0) where a and b are integers; also, in decimal form, any terminating or ultimately repeating decimal.

Ratios. A comparison between two things. For instance, someone can look at a group of people and refer to the "ratio of boys to girls" in the class. Suppose there are 35 students, 15 of whom are boys; the ratio of boys to girls is 15 to 20. See
http://www.purplemath.com/modules/ratio.htm

Real-life situations. Ways in which mathematical concepts are used in real life.

Real numbers. All numbers on a number line, including negative and positive integers, fractions, and irrational numbers.

Real-world applications. Ways in which mathematical concepts are used in real-life situations.

Rectangle. A four-sided polygon (a flat shape with straight sides) where every angle is a right angle (90°); opposite sides are parallel and of equal length.

Right triangle. A triangle that has one right angle (a right angle measures exactly 90°) – Only a single angle in a triangle can be a right angle or it would not be a triangle. A small square is used to mark which angle in the figure is the right angle.

Sets. A group or collection of things that go together (e.g., a group of four stars).

Side. In most general terms, a line segment that is part of the figure - it is connected at either end to another line segment, which, in turn, may or may not be connected to still other line segments.

Similar figures. Figures that have the same shape but different sizes.

Similar shapes. Objects of the same shape but different sizes in which the corresponding angles are the same.

Slope. The steepness/incline/grade of a line.

> ***Positive slope*** – the condition in which a line inclines from left to right.
> ***Negative slope*** – the condition in which a line declines from left to right.

Square. A four-sided polygon (a flat shape with straight sides) where all sides have equal length and every angle is a right angle (90°).

Square root. A value that can be multiplied by itself to give the original number (e.g., the square root of 25 is 5 because 5 x 5 = 25).

Square root notation. Numbers written using a radical √.

Subitize. To judge the number of objects in a group accurately without counting.

Three-dimensional geometric figures. The study of solid figures in three-dimensional space: cube, rectangular prism, sphere, cone, cylinder, and pyramid.

Two-dimensional figures. The study of two-dimensional figures in a plane; drawings of square, rectangle, circle, triangle, pentagon, hexagon, and octagon.

Unknown fixed quantities. A constant that is a quantity; a value that does not change.

Variable. A symbol for an unknown number to be solved; it is usually a letter like x or y (e.g., in x + 3 = 7, x is the variable).

Venn diagram. Made up of two or more overlapping circles. It is often used in mathematics to show relationships between sets. A Venn diagram enables students to organize similarities and differences visually.

Vertex (vertices, pl.). The point(s) where two or more edges meet (corners).

Volume. The amount of three-dimensional space an object occupies; capacity.

GLOSSARY OF SPECIAL EDUCATION TERMS

Accommodations. Changes in the administration of an assessment, such as setting, scheduling, timing, presentation format, response mode, or others, including any combination of these that does not change the construct intended to be measured by the assessment or the meaning of the resulting scores. Accommodations are used for equity, not advantage, and serve to level the playing field. To be appropriate, assessment accommodations must be identified in the student's Individualized Education Plan (IEP) or Section 504 plan and used regularly during instruction and classroom assessment.

Achievement descriptors. Narrative descriptions of performance levels that convey student performance at each achievement level and further defines content standards by connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the content standards. (See also "performance descriptors.")

Achievement levels. A measurement that distinguishes an adequate performance from a Level I or expert performance. Achievement levels provide a determination of the extent to which a student has met the content standards. (See also Performance levels.)

Achievement standard. A system that includes performance levels (e.g., unsatisfactory, Level III, advanced), descriptions of student performance for each level, examples of student work representing the entire range of performance for each level, and cut scores. A system of performance standards operationalizes and further defines content standards by connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the content standards. (See also "performance standards.")

Achievement test. An instrument designed to efficiently measure the amount of academic knowledge and/or skill a student has acquired from instruction. Such tests provide information that can be compared to either a norm group or a measure of performance, such as a standard.

Age appropriate. The characteristics of the skills taught, the activities and materials selected, and the language level employed that reflect the chronological age of the student.

Alignment. The similarity or match between or among content standards, achievement (performance) standards, curriculum, instruction, and assessments in terms of equal breadth, depth, and complexity of knowledge and skill expectations.

Alternate assessment. An instrument used in gathering information on the standards-based performance and progress of students whose disabilities preclude their valid and reliable participation in general assessments. Alternate assessments measure the performance of a relatively small population of students who are unable to participate in the general assessment system, even with accommodations, as determined by the IEP team.

Assessment. The process of collecting information about individuals, groups, or systems that relies upon a number of instruments, one of which may be a test. Therefore, assessment is a more comprehensive term than *test*.

Assessment literacy. The knowledge of the basic principles of sound assessment practice including terminology, development, administration, analysis, and standards of quality.

Assistance (vs. support). The degree to which the teacher provides aid to the student's performance that provides direct assistance in the content or skill being demonstrated by the student. That is, the assistance involves the teacher performing the cognitive work required. Assistance results in an invalidation of the item or score. (See also "support.")

Assistive technology. A device, piece of equipment, product system, or service that is used to increase, maintain, or improve the functional capabilities of a student with a disability. (See 34 CFR §300.5 and 300.6.)

Cues. Assistance, words, or actions provided to a student to increase the likelihood that the student will give the desired response.

Curriculum. A document that describes what teachers do in order to convey grade-level knowledge and skills to a student.

Depth. The level of cognitive processing (e.g., recognition, recall, problem solving, analysis, synthesis, and evaluation) required for success relative to the performance standards.

Disaggregation. The collection and reporting of student achievement results by particular subgroups (e.g., students with disabilities, limited English Level III students) to ascertain the subgroup's academic progress. Disaggregation makes it possible to compare subgroups or cohorts.

Essence of the standard. That which conveys the same ideas, skills, and content of the standard, expressed in simpler terms.

Essential Elements (EEs or CCEEs). The Common Core Essential Elements are specific statements of the content and skills that are linked to the Common Core State Standards (CCSS) grade level specific expectations for students with significant cognitive disabilities.

Grade Band Essential Element. A statement of essential precursor content and skills linked to the Common Core State Standards (CCSS) grade level clusters and indicators that maintain the essence of that standard, thereby identifying the grade-level expectations for students with significant cognitive disabilities to access and make progress in the general curriculum.

Grade level. The grade in which a student is enrolled.

Instructional Achievement Level Descriptors (IALDs). Describes student achievement and illustrates student performance. IALDs operationalize and further define Essential Elements by

connecting them to information that describes how well students are doing in learning the knowledge and skills contained in the Essential Elements.

Individualized Education Program (IEP). An IEP is a written plan, developed by a team of regular and special educators, parents, related service personnel, and the student, as appropriate, describing the specially designed instruction needed for an eligible exceptional student to progress in the content standards and objectives and to meet other educational needs.

Linked. A relationship between a grade level indicator for Common Core State Standards (CCSS) and Common Core Essential Elements (EEs or CCEEs) that reflects similar content and skills but does not match the breadth, depth, and complexity of the standards.

Multiple measures. Measurement of student or school performance through more than one form or test.

- For students, these might include teacher observations, performance assessments or portfolios.
- For schools, these might include dropout rates, absenteeism, college attendance or documented behavior problems

Natural cue. Assistance given to a student that provides a flow among the expectations presented by the educator, opportunities to learn, and the desired outcome exhibited by the student.

Opportunity to learn. The provision of learning conditions, including suitable adjustments, to maximize a student's chances of attaining the desired learning outcomes, such as the mastery of content standards.

Readability. The formatting of presented material that considers the organization of text; syntactic complexity of sentences; use of abstractions; density of concepts; sequence and organization of ideas; page format; sentence length; paragraph length; variety of punctuation; student background knowledge or interest; and use of illustrations or graphics in determining the appropriate level of difficulty of instructional or assessment materials.

Real-world application. The opportunity for a student to exhibit a behavior or complete a task that he or she would normally be expected to perform outside of the school environment.

Response requirements. The type, kind, or method of action required of a student to answer a question or testing item. The response may include, but is not limited to, reading, writing, speaking, creating, and drawing.

Stakeholders. A group of individuals perceived to be vested in a particular decision (e.g., a policy decision).

Standardized. An established procedure that assures that a test is administered with the same directions, and under the same conditions and is scored in the same manner for all students to ensure the comparability of scores. Standardization allows reliable and valid comparison to be made among students taking the test. The two major types of standardized tests are norm-referenced and criterion-referenced.

Standards. There are two types of standards, content and achievement (performance).

- **Content standards.** Statements of the subject-specific knowledge and skills that schools are expected to teach students, indicating what students should know and be able to do.

- **Achievement (Performance) standards.** Indices of qualities that specify how adept or competent a student demonstration must be and consist of the following four components:
    - levels that provide descriptive labels or narratives for student performance (i.e., advanced, Level III, etc.);
    - descriptions of what students at each particular level must demonstrate relative to the task;
    - examples of student work at each level illustrating the range of performance within each level; and
    - cut scores clearly separating each performance level.

Standards-based assessments. Assessments constructed to measure how well students have mastered specific content standards or skills.

Test. A measuring device or procedure. Educational tests are typically composed of questions or tasks designed to elicit predetermined behavioral responses or to measure specific academic content standards.

Test presentation. The method, manner, or structure in which test items or assessments are administered to the student.

Universal design of assessment. A method for developing an assessment to ensure accessibility by all students regardless of ability or disability. Universal design of assessment is based on principles used in the field of architecture in which user diversity is considered during the conceptual stage of development.

*Adapted from the *Glossary of Assessment Terms and Acronyms Used in Assessing Special Education Students: A Report from the Assessing Special Education Students (ASES) State Collaborative on Assessment and Student Standards (SCASS)*

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APPENDIX A

SEA/Stakeholder Demographics

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
Barbara Adams	IA	No response	K-12 Mathematics Curriculum Coordinator	No response	No response	Caucasian	21-25	PhD
Roula AlMouabbi	MI	Secondary Math 6-12; Bilingual Arabic/French 6-12	HS Bilingual Algebra/Geometry. College Algebra	9-11 and College	Arabic, French, African	Caucasian	21-25	MA
Robin Barbour	NC	All Subjects 4-6; 6-9 Math and Science; AIG certification	Secondary Math Consultant for NC Dept. of Public Instruction	7-8 Math; 9th Physical Science; Algebra 1; Integrated Math	General Education with inclusion experience	Caucasian	21-25	MA
Tamara Barrientos	MI	K-5 Elementary; 6-8 Math/Science	Director, Saginaw Valley State University Regional Mathematics and Science Center	6-8 Math	N/A	Hispanic	11-15	MA
DiRae Boyd	KS	Core Content Mesh K-6; Elementary K-9; LD K-9; MR K-9; SPED ELA K-9; SPED History and Government K-9; SPED Math K-9; SPED Science K-9	Functional 6-8 inter-related teacher	Special Education 6-8; Summer School to K-12 Special Education	MR; S/P; Autism; ED; DB; MD: HI; OHI; TBI; LD	Caucasian	16-20	BA
Lynda Brown	UT	ESL/Elem Math/Early Childhood Endorsement	Math Coach K-6 (4 schools, general and special ed.)	2-6 General Education	Special Education and Inclusion	Caucasian	30+	MED

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
Sue Burger	NJ	Elementary/ Teacher of Handicapped	Special Education/ Curriculum Specialist	HS Resource	HS Resource; Autism; OHI; MLD; BD; Preschool Disabled	Caucasian	30+	BA
Jennifer Burns	OK	Special Education – all contents	Assessment Coordinator for Special Education Services for State Dept. of Ed.	Special Education Pre-K and 6-8	S/P; MI/MO	Caucasian	6-10	MED/ MS
John Butz	IA	Math K-8; K-6 Elementary Education	2nd grade teacher	5th grade	Instruction of Special Education in General Education classroom	Caucasian	16-20	BA
Laurel Cakinberk	IA	Special Education Strategist II	Special Education	Middle/HS	MO/S/P	Caucasian	11-15	MA
Sharon Campione	MO	LD 1-8; MH/BD K-9; Spec Ed Admin K-12; Principal K-12	Functional, Life Skills, Self-contained 4-6	Middle School 7-8/Special Education	SSD Coordinator; Teacher Assist severe population	Caucasian	16-20	MS
Wendy Carver	UT	Communication Disorders/Special Education K-12+; Speech Language Pathology, Psychology, Mild/Mod Dis, ELA	Special Education Assessment Specialist	Special Education K-12+	MI/MO/S	Caucasian	30+	MS
Beth Cipoletti	WV	Math 7-12	Assistant Director, Office of	Math 7-12 and college; taught	Inclusion Classes	Caucasian	30+	EdD

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
			Assessment and Accountability	teacher preparation courses (mathematics)				
Emily Combs	MO	Math 5-9/ ELA 5-9	Math 7th grade	General Education Grade 6	Inclusion; special service, IEP	Caucasian	11-15	MS
Sidney Cooley	KS	Math; Special Education	State Mathematics Consultant	General Education 7-12	Integrated Math grades 7-9; State LD consultant	Caucasian	30+	PhD
Shirley Cooper	NJ	Math	State Mathematics Coordinator	General Education	Inclusion	African American	30+	MS
Jeff Crawford	WA	Math	HS Math, 9-12	College Mathematics	Low SES	Caucasian	16-20	MS
Amy Daugherty	OK	Special Education – All contents	Associate State Director for Special Education Services, State Dept. of Ed.	Special Education K-12	S/P; Emotional Disturbed	Caucasian	6-10	BS
John DeBenedetti	WA	Special Education	4-5 Extended Resource	N/A	Special Education teacher	Caucasian	6-10	BS
Thomas Deeter	IA	NA	Lead Consultant (General Education) Assessment, Accountability, Program Evaluation	General Education		Asian-Caucasian	21-25	PhD
Jennie DeFriez	UT	Administrative/ Supervisory Certification; Level	Utah State Office of Education Elementary Math	General Education Grades 4-7; Math/Science	Assistant to State Special Education Assessment	Caucasian	11-15	MED

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
		2 Math endorsement; Level 2 Elementary Education License, middle level education	Assessment Specialist/Assistant Special Education Assessment Specialist		Specialist			
Kirsten Dlugo	WA	6-8 ELA, Math, Reading and Special Education	Special Education Teacher 6-8, Life Skills Classroom	Ungraded classroom for blind ages 12-16	VI; DB; Aut; MD; LD; BD, ID	Caucasian	6-10	MED
Amber Eckes	WI	Elementary Education and LD; Reading Teacher	Special Education Manager Grades 6-8	Reading 6-8; Math 6-8 and summer classes K-3	Special Education manager/teacher	Caucasian	6-10	BS
John Eisenberg	VA	Special Education	Virginia Department of Education Director of Instructional Support and Related Services	Special Education	ASD; SD; ID	Caucasian	11-15	MS
Lin Everett	MO	K-5 Administrator/Principal; 4-8 SS; K-8 General Education: Lifetime Certificate; 4-8 Middle School Admin/Principal; Superintendent's certification, K-12	MO Dept. of Education Assistant Director of Assessment/Office of CCR	Self-contained 1-4; ELA Middle; Principal K-8, Methods for pre-service teachers/University	Special Ed Coordinator	Caucasian	30+	EdS
Dagny Fidler	IA	Director of Special	Vice-	Special Education K-	Focus on students	Caucasian	30+	PhD

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
		Education; PK-12 Principal; PK-12 Special Education Supervisor	Principal/Special Education Supervisor (focus on students with SCD)	12, College instruction	with significant disabilities			
Kim Fratto	UT	Under review	District Level Teacher Specialist for Students w/Significant Cognitive Disabilities	K-6 Special Education	K-6 Resource Teacher; Inclusion Specialist; Special Education Coordinator; Teacher specialist K-12+, Teacher Specialist, students with SCD	Caucasian	11-15	MS
Rosemary Gardner	WI	Elementary Education 1-8; SSLD PreK-12; Principal; Director of Special Education; Pupil Services	Special Education; Educational Programmer	General Education 1 & 2, and Special Education intermediate and middle school	Special Education Teacher/Support Admin	Caucasian	26-30	MS
Melissa Gholson	WV	Multi-Subjects K-8; Mental Impairments, Autism, Behavior Disorders, Specific LD K-21; Principal and Superintendent	WV Dept. of Education, Office of Asssessment and Accountability, Alternate Assessment and Accommodations	Elementary (general and special education), Middle School (special education); High School (general and special education), , College (teacher	Supervisor of Special Education; Special education teaching experience with autism, mild, moderate, severe and profound, mental	Caucasian	16-20	MA

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
				preparation courses)	impairments, behavior disorders, gifted and learning disabilities			
Debra Hawkins	WA	ESEA School Psychology	Director Classroom Assessment Integration	General Education Post-Secondary Level	Profoundly Mentally Handicapped	Caucasian	21-25	EdD
Linda Howley	MI	State Education Assessment Representative	State Education Assessment Representative			Caucasian	11-16	MS
Angelita Jagla	WA	Elementary K-8; Teacher of English as a Second Language; Reading and Math M.S. Ed; NBCT	General Education– 4th grade		Special Education, low SES, ELL	Mexican-American	6-10	MS
Brian Johnson	WI	Special Education	Special Education		CD; Autism; EBD	Caucasian	6-10	MS
MaryAnn Joseph	NJ	NBCT; Middle Childhood Generalist; Special Education K-12	Special Education Consultant NJDOE/OSEP	Special Education Severe/Profound, Middle School; 5-6 In Class Resource Planning (special ed), self-contained classroom ages 7-11; General and Special Education Pre-K-1	Severe/Profound; Learning Disabled K-8	Caucasian	30+	MED

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Sara King	MO	No response	Special Education ages 18-20	Special Education ages 14-20	Special Education	Caucasian	6-10	MA
Teresa Kraft	KS	Education of the Deaf	Curriculum and Assessment Coordinator, KS School for the Deaf		Deaf/HOH/Multi-handicapped; Visual Impairments	Caucasian	30+	MED
Tracey Lank	NJ	Special Education	Special Education 3-5 grades	Special Education 1, 2, and 6th grades	Multiple Disabilities	Caucasian	1-5	
Ronda Layman	NC	Speech Language; EC Administration	EC Lead Teacher/SLP-Autism and low incidence		Autism; Severe/Profound	Caucasian	21-25	MED
Wesley Lilly	WV	Special Education K-Adult (MI, LD, BD, Autism, Severe Mental Disabilities; Secondary Education; K-12 (Physical Education)	Secondary Special Education MI/Severe/Autism	Special Education K-8 MI/Severe/Autism/ LD/BD	MI/Severe/ Autism/LD/BD; worked with designing alternate assessment	Caucasian	6-10	MA
Diane Lucas	VA	Elementary Reading, Math, Social Studies, and Science	Special Education Classroom Resource Teacher (AT Team Leader)	Early Childhood Special Education	Special Education pre K-12, ID, SD, Autism, LD	Caucasian	30+	MS
Michele Luksa	KS	Severe Disabilities	Special Education Consulting Teacher for Elementary	Special Education Consulting Teacher 5-12	Severe Disabilities; Deaf-Blind, Autism	Caucasian	26-30	MA

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
Deborah Matthews	KS	Students with Significant Cognitive Disabilities and Early Childhood	Kansas State Department of Education	Early childhood-high school	Early Childhood; Students with Significant Cognitive Disabilities	Caucasian	21-25	MS
Melissa Mobley	WV	Autism/Mental Impairment	Supervisor of Special Education – Autism and all levels of mental impairment	Autism K-8	Autism; Mental Impairments preK-Adult	Caucasian	6-10	MA
Lisa New	WV	Math 7-12; Business Principles 7-12	HS Algebra I, Algebra support teacher	General Education Grades 5-12	Team teacher; inclusion; item writing for alternate assessment	Caucasian Native American	21-25	MS
Karen Pace	MO	Math 7-12	HS Math Teacher	General Education Math 7-9	LD, BD, ELL, low SES	Caucasian	30+	MED
Brain Pianosi	MI	Self-contained Elementary 6-8 Math/Science; K-12 Special Ed.; Cognitive Impairment Administration – certified elementary principal, supervisor and	Director of a Center-based school serving students with Moderate to Severe Cognitive, severe multiple impairments, autism; behavior needs	General Education 3rd grade; Special Education HS Cross Categorical	Deaf son; Daughter with LD; Special Olympics volunteer	Caucasian	21-25	MA

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		director certifications in special ed.						
Mary Richards	WI	WI Educator Grades 1-8	Math Coach PK-8	General Education K-6; Title I Math 1-4; Gifted and Talented Grades 1-5	Inclusion	Caucasian	30+	MS
Laura Scearce	VA	Math Specialist K-8	Math Coach K-5	Inclusion Grades 3 and 5	Inclusion; Gifted and Talented	Caucasian	11-15	MED
Lisa Seipert	UT	MI/MOD/Severe Special Education	ID/SID self-contained Grades 7-9	LD/CD Self-contained Grades 7-9	LD/ID/SID	Caucasian	11-15	BS
Katie Slane	NJ	Math and LA	7th Grade Special Education, self-contained and inclusive	Special Education 2-5 self-contained	LD and Autism	Caucasian	1-5	BA
Janet Sockwell	NC	Severe/Profound K-12; Mentally handicapped K-12; B/E Handicapped K-12; LD K-12; Birth - Kindergarten	Special Education Preschool Coordinator and Support for ID-Mod/Severe	Special Education K-12 moderate to profound	Moderate/severe/profound, behavior-emotional disturbed, pre-school	Caucasian	21-25	BS

Name	State	Area of Certification	Current Assignment	Other Grades Taught	Special Population Experience	Ethnicity	Years of Experience	Highest Degree
Christie Stephenson	OK	MI/Mod; Severe/Profound	Elementary Special Education Supervisor	K-12	LD. ID. MD Autism, OHI	Caucasian	6-10	BS
Deena Swain	WV	Multi-subjects K-8; BD; autism/admin	RESA Director of Special Education	General Education K-8; Math and Science at Alt. School/Juvenile Detention Center Grades 7-9; Autism K-12	Experience teaching students with ASD, Trainer of teachers and administrators on SE issues	Caucasian	16-20	MA
Emily Thatcher	IA	K-12 Strat I MD; K-12 Strat II MD. Multi-cat 6-12; BD K-6; Severe and Profound K-12; Special Education Consultant	Iowa Dept. of Ed., Bureau of Student and Family Support Services (SPED), Instructional Content Resource and Alternate Assessment Consultant	Special Education and Art K-12	22 years varied experience	Caucasian	21-25	MED
Larry Timm	MI	Special Education CI; Industrial Education	Middle School CI Math 6-8	General Education 6-8 Tech Ed.	Mod to Mild C.I.	Caucasian	16-20	MA
Mona Tjaden	KS	Elementary K-9; EMR and TMR Special Education K-9; Special Education Supervisor K-12; Library Media K-12	Special Education Program Coordinator	Special Education Program Coordinator	Special Education Teacher and Coordinator	Caucasian	30+	MS

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Janice Tornow	WA	General and Special Education K-12	WA Office of Superintendent of Public Instruction	Special Education K-12	Special Education Teacher and Administrator	Caucasian	30+	MED
Jane VanDeZande	MO	ELA and Special Education (Handicapped Learner)	Director of Assessment	5-8 Speech and Language and LD; ELA and Social Studies 9-12	Chapter I Director Math and Reading, Special Education	Irish American	16-20	Other Degree
Joyce Viscomi	VA	Elementary K-5 (reading, math, social studies, science)	Special Education Intellectually Impaired, Multiple Handicapped and OHI	Special Education – preK-12	Special Education Intellectually Impaired, Multiple Handicapped, Severe and Profound, OHI	Caucasian	21-25	BS
Nicole Warren	UT	Early Childhood Education; Elementary Math Endorsement; ESL Endorsement, Admin. Certification	Elementary Math Coach; General and Special Education, facilitate elementary endorsement classes	General Education Kindergarten. Coached all grades K-6.	Assisted Special Education Teachers in Math Curriculum, Instruction, and Assessment	Caucasian	11-15	MED
Roslynn Webb	VA	History/ELA	Math 6-8		Multi/Intellectual Disabilities	Black	6-10	MS
Deborah Wickham	VA	Postgraduate Professional License Admin PreK – 12; Early Education NK-4, Division Superintendent	Math Specialist K-5	General Education K-5 and college (per-service and graduate)	Worked with special needs students	Caucasian	26-30	PhD

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		License						
Joanne Winkelman	MI	Elementary and Special Education	State Agency	General Education 6-12	Special Education experience	Caucasian	21-25	PhD
Jeff Ziegler	WI	Math 9-12	HS Math Resource Teacher		Inclusion	Caucasian	16-20	MS